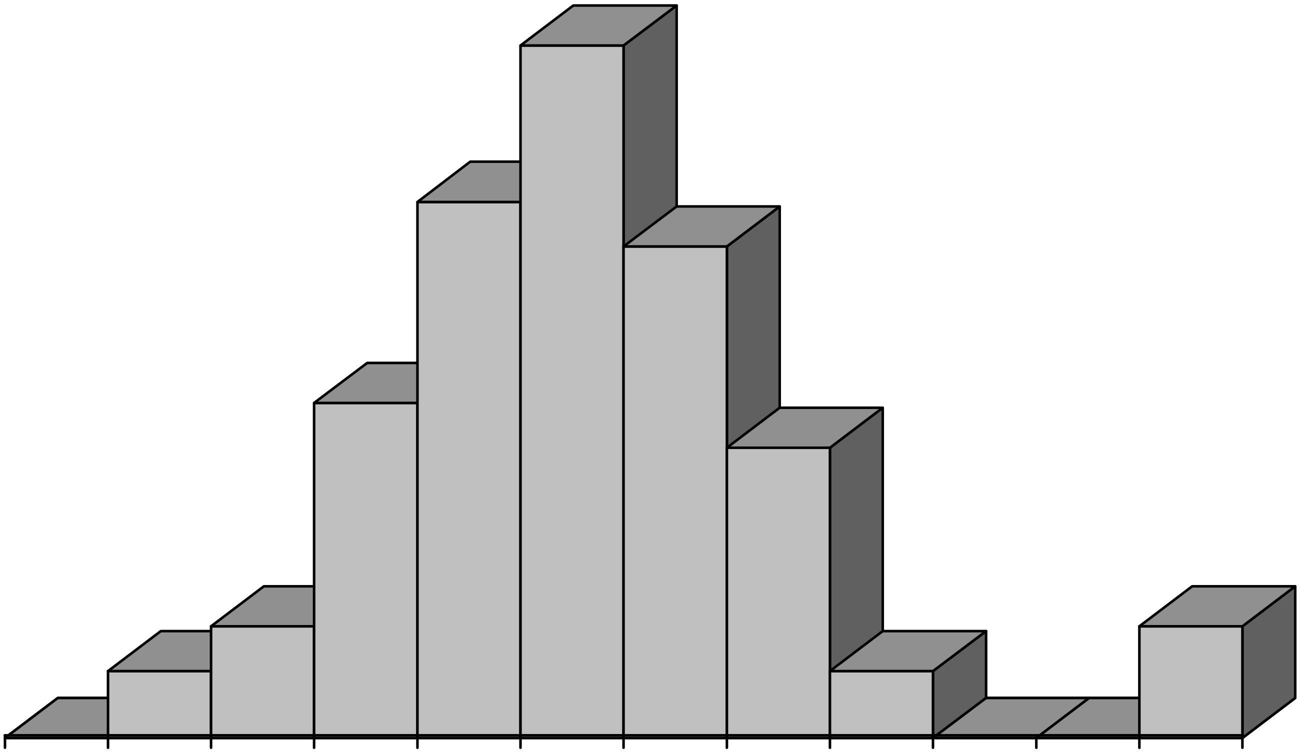
Idaho
Ratio Study Manual



2007 - 2008

TITLE PAGE

The 2007 - 2008 IDAHO RATIO STUDY MANUAL has been prepared by Alan Dornfest, Tax Policy Supervisor.

This manual supersedes any previous ratio study manual and is in effect beginning June 1, 2007.

2007 - 2008 IDAHO RATIO STUDY MANUAL

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INTRODUCTION

INTRODUCTION TO RATIO STUDY PROGRAM

Annual studies of the ratio between the market value determined by verified sales prices of real property and the assessed value of the same real property as stated on county assessors' rolls are conducted. These studies provide technical assistance to counties, test the results of the continuing appraisal process, and assist the State Tax Commission in its task of equalizing and certifying county property values. The ratio study is also used to certify adjusted market value for school equalization and levy purposes.

This manual explains procedures involved in ratio studies. Areas discussed include the following:

1. Sampling procedure, including sales verification process;
2. Types of studies;
3. Statistical analysis of data;
4. Education program and technical assistance;
5. The ratio study as an appraisal tool;
6. Standards and equalization procedures;
7. Definitions.

Examples are shown to help clarify the statistics presented and there is a section demonstrating how the ratio study can be used in the appraisal process. Historical background is also presented. The current manual attempts to correct terminology conflicts by adopting standard IAAO terminology whenever possible. Many of the changes in this manual are based on suggestions received during ratio study classes and committee meetings with assessors and appraisers. This interaction has been helpful and should continue so that future manuals will further improve.

The reader will find general information as well as complex formulas. It is far more important to understand the concepts presented so that the ratio study can be used in appraisal work.

Ratio Study System in Place 2007 - 2008

Categories to be analyzed in the 2007 ratio study will follow a new model. Beginning with this study only primary categories as defined in Rule 130 will be tested.

As was instituted in 2006, the median and median confidence interval will continue to be used to test compliance with ratio study standards. The 2007 ratio study, to be completed in 2008, marks the first change in Idaho ratio study analysis categories in several years. For the 2007 county ratio studies, the following system changes will be implemented:

- Categories to be tested have been redefined and will include the following:
  - Improved Residential (including manufactured homes on same ownership land);
  - Unimproved Residential;
  - Improved Commercial;
  - Unimproved Commercial;
  - Manufactured homes on leased land.
- If equalization adjustments are warranted, such adjustments would be applied to any component category for which at least one observation is included in the sample (*see Rule 131 for exceptions*).
- 90% (two-tailed) confidence intervals will still be used initially to test compliance. We will continue to test compliance with lower (80%) confidence intervals around the median when the sample median is ***not*** within the 90% - 110% range for two consecutive ratio studies. However, no category will be found to be out of compliance using this lower degree of confidence in the 2007 or 2008 ratio studies.

School District Ratio Studies:

- The weighted mean ratio will be used, except when distorted by non-representative ratios. In this instance, the median will be used.
- Equalization adjustments will only be considered when the appropriate (weighted mean or median) confidence interval fails to include 95% or 105%.
- School district ratio studies will be conducted for the Boise School District only.

Additional information concerning this procedure is found in rules 130 and 131, found in Appendix VII, and in the "Standards and Equalization Procedures" section of this manual.

Manual vs. Rules

This manual is intended to provide information for training and technical assistance. Compliance standards stated in this manual are advisory in nature, unless specifically incorporated into State Tax Commission rules. Statutorily set ratio study guidelines for school district ratio studies can be found in Idaho Code §63-315. The requirement for equalization of categories of property by the State Tax Commission is found in Idaho Code §63-109. Assessment level compliance standards are set by State Tax Commission rule 131, while school ratio study procedures are described in rule 315.

HISTORICAL BACKGROUND

To better understand the ratio study, it is important to understand some of the history of the assessment process in Idaho and how equalization has fit into this process. This section is intended as an undocumented outline of the past 40 years, designed to give a broad, general overview of this period.

Evolution of the Valuation Process

In reviewing the valuation process over time, several distinct periods appear to exist. These can be summarized as follows:

Pre - 1965:	Counties establish different assessment ratios (levels) independently and without state direction.
1965:	The legislature mandates ratios of 20% for locally assessed real and personal property and 40% for centrally assessed operating property.
1967:	State Supreme Court rules classification (see 1965) unconstitutional and requires all property to be assessed at 20%. This level is to be phased in by 1979 (later revised to 1982).
1967-1978:	Declared ratios of each county approach 20%; actual ratios lag further behind each year, with the lowest ratios found in residential categories.
1978-1980:	1% Initiative passes with requirement that property be assessed at full market value as of December 31, 1978; the reassessment is to be completed in time for the 1980 rolls. This results in a typical residential level of about 80% of market value.
1981-1982:	For 1981, the December 1978 values are to be increased by 4.04% (2% for 1979 and the same for 1980). Full ***current*** market value is to be achieved for 1982.
1982-1989:	Through 1987, current market value is established each year, based on sales centering one year prior to the lien date. Beginning in

	1988 sales data centers 6 months prior to the lien date (prior calendar year).
1989-1991:	Current market value each year is still required. Prior calendar year sales are used to test and provide information for ongoing appraisals.
1992-present:	Current market value is still required, with prior year sales used for ongoing (county) appraisals, but the last three months of the prior year and the first nine months of the current year are used in the ratio study to test assessment conditions.

Evolution of the Ratio Study and Equalization Program

The ratio study and its use changed and grew during this same period. Changes can be outlined as follows:

Pre - 1960:	There were sporadic studies, with a partial study in 1955 and a full study of each county in 1958. Use is unknown.
Mid 60's - 1978:	Annual studies used for school equalization purposes. Equalization based on county-wide weighted average assessment level, restricted to school funds only; did not equalize inequity between categories.
1979 - 1981:	Transition to current system; no equalization.
1982 - 1987:	State ordered trending by category if out of compliance with level standards.
1988 - 1991:	State ordered trending only if category out of compliance for two successive ratio studies.
1992 - 1994:	Compliance determined with burden of proof of non-compliance on State Tax Commission. Two successive non-compliance studies required to produce a trending order. School equalization reinstated beginning with 1993 ratio study.

1995 - present: Equalization orders may result after just one year of non-compliance. Special follow-up studies will test current (not just past) year's assessment conditions prior to final compliance determination. Beginning with 2002 ratio study, burden of proof of non-compliance lowered given two consecutive years with sample measures of level outside acceptable range.

RATIO STUDY SAMPLING PROCEDURE

For the most meaningful ratio study, sales information must be collected, confirmed, and verified to prepare samples of arm's length, market value transactions which are representative of each category or type of property to be studied. The ideal sample would be:

1. Randomly selected;
2. Proportionally representative of all locational influences and pertinent property characteristics;
3. Of sufficient size to be considered reliable.

Since there presumably is not an equal opportunity for each property to sell, provide information, and be in the ratio study, the randomness test is not met. Similarly, requirements 2 and 3 are somewhat uncontrollable. Therefore, to maintain some degree of statistical validity, sample size goals should be as follows:

1. Obtain the greatest possible number of acceptable sales;
2. Do not exclude any sales unless verifiably invalid or if these sales over-represent certain properties;
3. Make additional efforts to obtain sales in areas and categories which, traditionally, have few verified transactions;
4. Check for over-representation of "hot spots". Do not allow these areas to contain more sales than the proportion of property in the "hot spot" to the category being tested. Note that the "hot spot" designation also applies to over-representation of certain value related property characteristics. For example, if 20% of the residential improvements have more than 2500 square feet of living area, and this group accounts for 50% of the ratio study sample, sales should be removed randomly until the correct proportions are achieved.

Because of our inability to randomly sample property, we cannot truly estimate the number of sales necessary to produce a reliable and valid ratio study. However, it should be noted that the major factors which influence sample size requirements are:

1. Uniformity: Fewer sales are needed to study areas with good assessment uniformity.

2. Acceptable error: If a larger error in results is considered acceptable, a smaller sample size is indicated.

The number of parcels in an area or category has only a minor influence on sample size. The following examples use standard statistical sample size formulas to demonstrate these influences:

	Example A	**Example B**	**Example C**
Total # of parcels	5,000	5,000	500
Standard deviation: (low numbers = good)	15%	25%	25%
Acceptable error	± 10%	± 5%	± 5%
Required sample size:	9	98	84

The procedure used to derive sample size in these examples assumes that a random sample of any size can be produced. Because this is not true in ratio study sampling, sample size formulas are of only marginal use in establishing ratio study sample sizes.

The primary source of sales information will be the deed records of the county. All open market sales which are not to be excluded as invalid, as indicated below, should be included in the ratio study, unless such inclusion can be demonstrated to produce over-representation of certain value influences. Validity of sales data should be determined by confirming the details of each transaction.

Confirmation may be made by contact, in person or by mail, with either the grantee, the grantor, or other knowledgeable person who is fully informed of the terms of the transaction. Sales may also be confirmed by review of sales documents. These documents include:

1. Purchase agreements,
2. Escrow documents, and
3. Broker records.

When any portion of the property studied is exempt from property taxes, the sale must be adjusted. For homeowner's and hardship exemptions, the exempted value should be added back to the taxable value of the property before the ratio is calculated. For all other exemptions, each sale price should be adjusted to remove the exempted value before the ratio is calculated. If the adjustment cannot be calculated, the sale should be deleted from the study.

A sample which includes personal property may be used in the ratio study if the selling price can be adjusted satisfactorily to eliminate the personal property value. When such adjustments are considered, the amount to be subtracted from the sale price should be market derived and should not merely be the cost of the personal property.

The following sales situations are presented to illustrate the types of transactions usually presumed to be ***unsuitable*** for use in an assessment ratio study:

1. The deed does not show warranty of title by the grantor; however, other types of deeds may be used in the ratio study if verification proves they are bona fide transactions.

2. The subject of the grant is a partial interest.

3. The grantee or grantor is a federal, state, county, municipality or other political subdivision, or is a public utility.

4. The grantee is a bank, insurance company, building and loan association, or other financial institution.

5. The grantee is a charitable, educational, or religious institution.

6. The grantee and grantor are related by blood or marriage, or are corporate affiliates. Sales between business associates should be carefully screened.

7. The grantee and grantor are the same and the deed is a convenience to change the nature of the interest in the property. (Example: Tenancy in common to tenancy by the entireties.)

8. The subject property constitutes or is a part of a trade or exchange of properties.

9. The grantor is transferring property to avoid a lien or judgment.

10. The sale results from judicial order, decree, or proceedings, and grantor is a sheriff, receiver, or other court officer.

Sales that can be clearly identified as falling into any of the above categories are considered potential candidates for rejection from the ratio study.

It is important to note that sales between relatives should ***not*** automatically be excluded. Often, sales prices are ***not*** demonstrably influenced by family relationships and this effect should be determined when verifying sales.

The ratio that is calculated for any sale should not be used as an indication of validity. However, studies have shown that extreme outlier ratios (very high or very low) often indicate doubtful sample validity. Since outliers can substantially impact measures of assessment level in small samples, non-typical ratios should be used as flags to identify sales which should then be subject to additional verification. As a rough rule of thumb, sales with ratios outside of a range of ±2 standard deviations around the mean should be reviewed. It is no longer permissible to use an automatic 5% exclusion procedure. Alternate procedures to be employed are based on the inter-quartile range and can be found in the IAAO *1999 Standard on Ratio Studies*.

Example 1: Outlier Review Guide

Sale #	Assessed Value	Sales Price	Ratio (%)
1	15,000	25,000	60.00
2	15,000	22,000	68.18
3	17,000	20,000	85.00
4	19,000	22,000	86.36
5	25,000	27,000	92.59
6	24,000	25,500	94.12
7	25,000	25,000	100.00
8	20,000	16,000	125.00
9	35,000	25,000	140.00
10	55,000	25,000	220.00

Although the assessment level appears to be acceptable in this example, the mean is 107.13% and uniformity shown is very poor (COD = 30.74%). The standard deviation is 46.22%. Sale #10 exceeds the mean by more than 2 standard deviations and should be reviewed. If sale #10 were found to be invalid, the mean would become 94.6% and the standard deviation 25.2%.

The procedure for rejecting sales is:

1. Sales to be included are submitted by the county assessor to the consulting appraiser. The consulting appraiser may determine that additional sales are needed and may search for and include these when possible.

2. The consulting appraiser and the county assessor should review the sales and the consulting appraiser may use discretion to delete invalid sales found in the study. If there is disagreement between the consulting appraiser and the assessor, the assessor

should make written recommendation to the State Tax Commission regarding sales to be added or eliminated.

3. The State Tax Commission decides whether to follow the county's recommendations and notifies the county and consulting appraiser accordingly.

Contracts for sale are usable in the ratio study if the conditions of the sale meet the requirements of a bona fide, arm's length transaction. Implicit in this term, arm's length transaction, is the consummation of a sale as of a specified date and the passing of title from seller to buyer under conditions whereby:

1. buyer and seller are typically motivated.
2. both parties are well informed or well advised, and each acts in what is consider their own best interest.
3. a reasonable time is allowed for exposure in the open market.
4. payment is made in cash, or with financing which is on terms generally available in the community at the specified date and typical for the property type in its locale.
5. the price represents a normal consideration for the property sold and is unaffected by special financing amounts and/or terms, services, fees, costs, or credits incurred in the transaction.

RULES

The basic guidelines and standards for the ratio study are contained in the following State Tax Commission rules:

Rule 217:	Rule pertaining to market value and appraisal.
Rules 314 & 316:	Rules pertaining to reappraisal program and compliance.
Rule 130:	Rule listing and describing primary property categories.
Rule 131:	Rule pertaining to use of ratio study in equalization.
Rule 315:	Rule pertaining to school district ratio studies.

TYPES OF STUDIES

The type of ratio study varies with intended use. Studies may be used for many purposes, including:

1.) determining assessment conditions for property of a particular type or class, or at a particular location;

2.) establishing baseline conditions prior to reappraisal and monitoring the progress of reappraisal work;

3.) equalizing property values to ensure equal treatment by category and equal effect of exemptions;

4.) computing adjusted market value for school equalization and levy purposes.

Depending on intended use, the time period from which sales to be included are to be drawn will vary, and assessments against which these sales are to be compared will also vary.

Studies by Counties

Studies done by local officials will generally relate to purposes (1) and (2) shown above. These studies should, ordinarily, involve sales occurring during the calendar year immediately preceding the assessment year. For example, for year 2008 assessments, calendar year 2007 sales should be used. However, this should not be considered to be inflexible. If sufficient sales are not available during one year, it is permissible to extend the sales period, provided that proper, documented time adjustments are developed and that economic conditions have not become greatly altered.

Sales data for 2007 (or a longer period, if necessary) could be compared to either 2007 or 2008 assessments, depending on whether the study was designed to determine initial (baseline) or final assessment conditions for 2008. Comparison to 2007 assessments would also represent a final review of 2007 assessments, and would help the assessor to determine the accuracy and validity of decisions made and data used for that year.

For every period of sales used in a ratio study, time adjustments must be considered. When using the prior calendar year's sales, the sale prices typically will reflect market values as of July 1 of the prior year. Since the assessment date is six months later, sale prices should be time adjusted forward to reflect value as of January 1 of the assessment year. Procedures to use to determine appropriate time adjustments are found in the IAAO **Property Appraisal and Assessment Administration** (**PAAA**) textbook.

Special studies reflecting various geo-economic areas, and classes or types of properties are strongly recommended. Although the county is required to appraise all property at least once every five years, special emphasis should be focused on those areas which have poor uniformity as demonstrated by ratio studies.

2007 County Equalization Ratio Study

Each category of property must be in compliance with assessment level standards each year. County equalization ratio studies test compliance and are authorized under Idaho Code §63-109 and State Tax Commission rule 131. The 2007 final county ratio study is by category of property based on sales and assessments as follows:

> Sales occurring between Oct. 1, 2006 and Sept. 30, 2007 are adjusted for time (to January 1, 2007) and compared to 2007 assessments. This study will be completed in March, 2008 and is considered a final report on 2007 assessment conditions.
>
> Time adjustments must be considered and made whenever provable in the market. Different adjustments may be necessary to reflect different amounts of appreciation in different categories of property.
>
> Counties will be notified of any category that is out of compliance (see: "Standards and Equalization Procedures" section). Results that are out of compliance trigger a follow-up study which will compare 2008 assessments with sales occurring between January 1, 2007 and December 31, 2007. For this follow-up study, sale prices will be time-adjusted to January 1, 2008.
> Any category that is not considered in compliance after completion of the follow-up study will be reported to the State Tax Commission, which may take equalization action at its August, 2008 meeting. The State Tax Commission may delay implementation of any equalization adjustment for one year, if there is reason to question the representativeness of the original ratio study.

Statistically, the burden of proof of noncompliance is on the State Tax Commission as the equalizing agency. Except when sample level measurements remain outside the 90% - 110% range for two consecutive years, a conclusion of non-compliance will be reached only when the State Tax Commission is at least 95% certain that the mean (if normally distributed) or median (if not normally distributed) level of assessment is **not** between 90% and 110% of market value for a given category of property (see: "Standards and Equalization Procedures" section for more complete explanation).

Perspective on the Ratio Study & Equalization

The use of the ratio study as outlined in this section conforms with major features of the IAAO *1999 Standard on Ratio Studies* (although no provision in the Idaho system tests for differences between categories of property in a county - the IAAO *Standard...* suggests no more than a 5% permissible difference). This *Standard...* advocates use of sales spanning or after the assessment date for equalization purposes. The use of sales following the assessment date creates an independent check of assessed values and lessens the need for additional monitoring (checking assessment rolls, etc.) to confirm ratio study results. The equalization ratio study becomes simply an audit procedure to determine compliance and the need for equalization adjustments. This particular study is ***not*** designed to assist in the appraisal process or to otherwise provide technical assistance to the county. Those functions are met by ratio studies done locally or with State Tax Commission assistance, based on sales from an earlier time frame.

Ratio study sampling procedures rely on sales which may not occur in random patterns and, thereby, may not conform with standardized statistical survey sampling procedures which enable precise calculation of reliability. For any statistical validity in both equalization and reappraisal ratio studies, sample representativeness is critical. If, for example, a new area begins to sell after the assessment date, an influx of sales from this area may cause over-representation of assessment conditions which differ significantly from those in the remainder of the category. Follow-up studies on non-complying categories of property will aid in ascertaining assessment conditions and will more nearly reflect value changes made by assessors for the current year subject to equalization.

Rural Area Equalization

The most difficult areas to appraise or equalize are those with highly erratic markets or with few sales. The current ratio study standards greatly reduce the possibility of equalization adjustments in categories with poor reliability due to small samples or poor uniformity, because the burden of proof of noncompliance is on the State Tax Commission. Under current standards, even in the largest county or category, ***every*** category with a median between 90% and 110% of market value is considered in compliance. Categories represented by small, poorly reliable samples are allowed considerable deviation from this range, as expressed by the sample median (known in statistics as a point estimate). For example, in the first year of testing, a sample of 10 sales with median of 85% and a median 90% confidence interval of 76% - 92% would not be considered out of compliance (on the low side). After two years with a sample median below 90% however, an 80% confidence interval would be required to overlap the 90% - 110% range for the category to be considered in compliance. This is more stringent, since 80% confidence intervals are nearly always narrower than 90% confidence intervals. Additional charts and information on compliance are presented in the "Standards and Equalization Procedures" section.

Often, due to limited sales in sparsely populated areas, certain categories of property have assessment conditions determined by analysis of a small number of sales. There is no provision in Idaho law to preclude equalization if a particular minimum number of sales is not available. However, as long as this manual is in effect, State Tax Commission staff will not recommend State Tax Commission ordered value adjustments if the sample size is less than five. The analysis of any sample with fewer than five sales is intended only as a guide to the assessor. The State Tax Commission may, at its discretion, add appraisals conducted by Commission staff to small samples to improve representativeness and attain minimum sample size for equalization studies.

The importance of maximizing the amount of sales data to enhance study reliability cannot be over-emphasized.

Ratio Study use in School Equalization

Idaho Code §63-315 imposes a requirement for the State Tax Commission to compute adjusted market value for the Boise School District and to publish the statistical measures computed in the ratio study done to fulfill this requirement.

Procedure

The weighted mean ratio is used for computing the Boise School District's adjusted value unless distortion can be proven, in which case the median is substituted. The compliance range is 95% - 105% and there will no adjustment to category values provided a 90% confidence interval around the weighted mean (or median, if appropriate) overlaps this range. This ratio study is conducted by property "designation" instead of property category. All categories are to be assigned to one of two designations:

1. Residential, including manufactured housing or
2. Commercial.

A complete discussion of procedures to be used is found in Rule 315.

Discussion

Actual calculation of school district adjusted market values is done using spreadsheet software. Copies of all calculations will be made available electronically on request.

Statistical Measures

Statistical measures are computed using the sales and appraisal samples described earlier in this report. All measures are computed in accordance with standard statistical procedures described in this manual and in the IAAO *1999 Standard on Ratio Studies.*

STATISTICAL ANALYSIS OF DATA

In conducting assessment ratio studies the State Tax Commission performs statistical analysis of all verified sales and appraisals to be used. The results of this analysis are compiled and presented on tables, which identify the area being studied and category(ies) included (see Appendix I).

Results shown include statistical measures of central tendency (level), variability (uniformity), and reliability (precision). Tests of assessment progressivity/regressivity and of the normality of the distribution of assessment ratios are included whenever possible. Worksheets demonstrating procedures for many of these statistical tests can be found in Appendix III.

MEASURING ASSESSMENT LEVEL

Measurements which determine assessment level do so by establishing what is known statistically as the central tendency of the observations (in this case, the ratios). The goal is to determine one number which best represents assessment level. The number is based on the available sales data and is computed from ratios found by dividing each assessed value by the sale price of that property.

Assessment ratios can be expressed in decimal or percent form and are calculated in the following way:

$$\text{ratio} = \frac{\text{Assessed Value}}{\text{Sales Price}}$$

The term, **A/S** also means "ratio".

Example 2:

Assessed Value = $40,000
Sales Price = $50,000

$$\text{ratio} = \frac{\$40{,}000}{\$50{,}000} = 0.80 = 80\% = \text{A/S}$$

Using a ratio of 100% as a proxy for market value and as the primary point of reference, ratios will always fall into 3 groups:

(perfect) A. ratio = 100%; Assessed Value = Sales Price

(low) B. ratio < 100%; Assessed Value < Sales Price
(< means less than)

(high) C. ratio > 100%; Assessed Value > Sales Price
(> means greater than)

Although the ideal ratio is 100%, in practice it is rarely possible to precisely predict the selling price of individual properties. This prediction is further complicated by the fact that most properties are not currently for sale.

In mass appraisal aiming at market value, we expect approximately equal numbers of properties to sell for more than, or less than, their assessed values. Appraisal errors or marketplace uncertainty should be random. In other words, if the goal is market value (100%), individual properties randomly should be expected to appear to be assessed too high or too low, but a category as a whole, measured by a sufficient number of representative sales, should appear to be assessed close to 100%.

In Idaho, we calculate four different tests of assessment level. The purpose of each is to discover whether differences between assessments and sale prices are random, individual events or are systematic, resulting in low or high overall level of assessment. Four tests are used because each test is subject to different types of distortion or bias, the effect of which is minimized by reviewing the four results. The tests and their identifying symbols are:

1. Mean $\left(\overline{A/S}\right)$, also known as:
    a. Arithmetic Mean or simple Average
    b. Unweighted Mean or Unweighted Average
2. Median $(\widetilde{A/S})$
3. Geometric Mean
4. Weighted Mean $(\bar{A}/\bar{S})$, also known as: Sales Weighted Mean

Each of these measures is calculated on the sales or sales and appraisal ratios that constitute the sample. The results are point estimates or statistics related only to the sample. Additional tests of the reliability of these statistics are necessary to draw inferences about the population of unsold and sold properties that the sample is designed to represent (see "Statistical Measures of Reliability" section).

Mean $\left(\overline{A/S}\right)$:

The mean ratio is determined by summing the ratios computed for each sale and dividing this total by the number of sales in the sample. (See computation in Example 6 following.)

The mean has the advantages of being simple to compute and easy to understand. Another advantage is that it takes all measurements into account. It is also used as a basis for certain measurements of uniformity. However, a small number of very low or high assessment ratios tend to weigh heavily on the mean, distorting it (usually on the high side) so that it is often not the truest measure of assessment level. In fact, because of the mathematics involved in ratios, the mean is biased on the high side, although this is not always apparent. In small samples this distortion tends to be more pronounced. There is also a tendency to overuse this statistic. It is important to realize that a mean of 100% does not necessarily mean good assessment conditions.

Geometric Mean:

The geometric mean is a measure of assessment level which is not as susceptible to influence from a few extremely high ratios as the arithmetic mean. It does not, however, correct for problems caused by low ratios and will never be higher than the arithmetic mean. The geometric mean also suffers from being more complex and therefore less understandable. Finally, there are no corresponding measures of reliability to test the precision of this statistic. Example 6 shows the geometric mean for a sample and presents a comparison to the mean.

Geometric Mean:

A measure of level determined by multiplying all of the ratios in a sample together and then taking the "n"th root of the product of this calculation.

geo. mean = $(A_1/S_1 * A_2/S_2 * A_3/S_3 *\ldots A_n/S_n)^{1/n}$

where A_n/S_n represents each ratio in the sample;
and n = the number of ratios in the sample.

Median $(\widetilde{A/S})$:

The median ratio is an indicator of the central ratio in any sample. It is determined by arraying all of the ratios from a particular category and finding the midpoint. Again, it is possible to compare this value to both the arithmetic and geometric means, with large differences indicating problems in either sampling or county appraisals. The median is also used in calculating the coefficient of dispersion, discussed under the uniformity heading. The median is considered an unbiased estimator of level, since it is not subject to the effects of outlying ratios; however, this can be a disadvantage as well as an advantage, since valid outliers are not reflected.

Once the ratios are computed and arrayed, the rank or order number corresponding to the median ratio can be found from this formula:

median rank = .5(n) + .5,
where n = the number of sales in the sample.

Example 3:

Sale #	Ratio
1	80%
2	85%
3	90%
4	95%
5	100%

Median rank = .5(5) + .5 = 3
The third ratio is 90% and this is the median.

Example 4:

Sale #	Ratio
1	80%
2	85%
3	90%
4	95%
5	100%
6	105%

Median rank = .5(6) + .5 = 3.5

In Example 4, the median ratio is between the third and fourth ratios. These two ratios are added together with the sum averaged (divided by 2) to compute the median:

ratio #3 : 90%
ratio #4 : 95%
sum : 185%

sum / 2 : 185% / 2 = 92.5%

The median is 92.5%

(Note: ( ) adjacent to a number indicate that the number is to be multiplied by whatever is inside the ( ).)

Weighted Mean ($\bar{A}/\bar{S}$):

The weighted mean differs from the mean in that the computation is based on the total assessed value for the entire sample divided by total of all sales prices for all sales in the sample. (See computation in Examples 7 and 8 following.)

In the determination of this statistic, sales of more expensive property weigh more heavily and exert more influence on the result than those of less costly property. Outlying individual ratios do not exert strong influence on this statistic, but cautious use is recommended, since value weighting may cause considerable distortion, particularly by very high value occasional sales which may have non-representative low ratios.

Examples comparing the mean, median, and weighted mean follow:

COMPARISON OF THREE MEASURES OF ASSESSMENT LEVEL:

Example 5: WTD. MEAN = MEAN = MEDIAN

Sale #	Assessed Value	Sale Price	Ratio
1	$ 20,000	$ 50,000	40.00%
2	30,000	50,000	60.00%
3	40,000	50,000	80.00%
4	50,000	50,000	100.00%
5	60,000	50,000	120.00%
6	70,000	50,000	140.00%
7	80,000	50,000	160.00%
Totals:	**350,000**	**350,000**	**700.00%**

MEAN = 100.00%
MEDIAN = 100.00%
WTD. MEAN = 100.00%

Measures of level are considered biased if they tend to distort the impression of the true assessment level. In Example 5, the three measures are equally useable with distortion caused only by different assessments of properties treated identically in the market.

COMPARISON OF MEASURES OF ASSESSMENT LEVEL:

Example 6: MEAN > MEDIAN & SALES WTD. MEAN

Sale #	Assessed Value	Sale Price	Ratio
1	$ 80,000	$ 50,000	160.00%
2	75,000	60,000	125.00%
3	70,000	70,000	100.00%
4	65,000	80,000	81.25%
5	60,000	90,000	66.67%
6	55,000	100,000	55.00%
7	50,000	110,000	45.45%
Totals:	**455,000**	**560,000**	**633.37%**

MEAN =	90.48%
MEDIAN =	81.25%
WTD. MEAN =	81.25%
GEOM. MEAN =	82.98%

COMPUTATIONS:

633.37/7 = 90.48%	(mean) (the “/” means to divide)
.5 x 7 + .5 = 4 = 81.25%	(median)
455,000/560,000 = 81.25%	(wtd. Mean)
160 x 125 x 100 x 81.25 x 66.67 x 55 x 45.45 = $(27081979031250)^{1/7}$ = 82.98%	(geometric mean)

In Example 6, the mean is distorted by high ratios. The best indicator of level is probably the median, with ratios exceeding this point by up to 80 points, while the lowest ratio is within 36 points. Note that in comparison to the mean, the geometric mean is affected to a much lesser extent by the high ratio sales.

COMPARISON OF THREE MEASURES OF ASSESSMENT LEVEL:

Example 7: WTD. MEAN >> MEAN & MEDIAN

Sale #	Assessed Value	Sale Price	Ratio
1	$ 10,000	$ 20,000	50.00%
2	20,000	40,000	50.00%
3	30,000	60,000	50.00%
4	60,000	80,000	75.00%
5	90,000	100,000	90.00%
6	120,000	120,000	100.00%
7	160,000	140,000	114.29%
Totals:	**490,000**	**560,000**	**529.29%**

MEAN =	75.61%
MEDIAN =	75.00%
WTD. MEAN =	87.50%

COMPARISON OF THREE MEASURES OF ASSESSMENT LEVEL:

Example 8: WTD. MEAN << MEAN & MEDIAN

Sale #	Assessed Value	Sale Price	Ratio
1	$ 40,000	$ 20,000	200.00%
2	60,000	40,000	150.00%
3	60,000	60,000	100.00%
4	75,000	80,000	93.75%
5	60,000	100,000	60.00%
6	70,000	120,000	58.33%
7	30,000	140,000	21.43%
Totals:	**395,000**	**560,000**	**683.51%**

MEAN =	97.64%
MEDIAN =	93.75%
WTD. MEAN =	70.54%

The weighted mean fails as a valid indicator in Example 7, where high ratios on higher priced property distort this measurement upwards. The opposite occurs in Example 8, where assessments are too high on lower priced property and too low on higher priced property. The weighted mean is questionable in this case as well.

The following chart provides a summary of assessment level statistics:

Assessment Level		
	Advantages	**Disadvantages**
Mean	Uses all data; basis for uniformity and reliability statistics	Biased high (affected more by high ratios)
Weighted Mean	Eliminates distortion due to high or low ratios	Price related weighting distorts toward ratios on higher priced property
Median	Unbiased (by extreme data)	Ignores all but 1 or 2 ratios; no predictive capability
Geometric Mean	Unbiased (by extreme high ratios)	Not useful as basis for uniformity and reliablility statistics

MEASURING ASSESSMENT UNIFORMITY

Uniformity determines the quality and inherent equity of property assessments. Although both the appraisal and the market transaction are subject to distortion on any individual property, if the magnitude of this distortion is consistently large, taxes paid by similar properties in the same area will differ widely. The goal of a fair assessment program is to reduce inequity of this type.

There are two overall types of inequity that can occur:

1. Inequity between categories.
2. Inequity within a given category.

In the first case, inequity results when the assessment level is lower in one category than another. This situation becomes apparent when level indicators from different categories are compared.

In the second case, the distortion is entirely within one category and is not indicated by measurements of level. The following chart illustrates this situation:

LEVEL VS. UNIFORMITY

Example 9:

Sale #	Assessed Value	Sale Price	Ratio
1	$ 10,000	$ 25,000	40.00%
2	30,000	50,000	60.00%
3	22,500	30,000	75.00%
4	60,000	60,000	100.00%
5	37,500	30,000	125.00%
6	70,000	50,000	140.00%
7	40,000	25,000	160.00%
Totals:	**270,000**	**270,000**	**700.00%**

MEAN = 100.00% *
* MEASURES
MEDIAN = 100.00% * OF
* ASSESSMENT
WTD. MEAN = 100.00% * LEVEL
*
GEOMETRIC MEAN = 90.68% *

(COD) COEFFICIENT OF DISPERSION = 35.71% * MEASURES
* OF
(COV) COEFFICIENT OF VARIATION = 44.06% * UNIFORMITY

Although all of the level measurements indicate that market value has been achieved on an overall basis, significant variation is present and will prevent many taxpayers from being taxed equitably.

If the seven sales shown accurately represent assessment conditions, each sale can be thought of as representing about 14% of the property. In other words, 14% of the property is assessed at 40% of value, 14% at 60%, and so on. The type of inequity demonstrated in Example 9 is known as **horizontal** inequity, because it occurs throughout a category of property and is ***not*** more pronounced on higher or lower priced property.

LEVEL VS. UNIFORMITY

Example 10:

Sale #	Assessed Value	Sale Price	Ratio
1	$ 21,000	$ 25,000	84.00%
2	44,000	50,000	88.00%
3	28,000	30,000	93.33%
4	60,000	60,000	100.00%
5	32,000	30,000	106.67%
6	56,000	50,000	112.00%
7	29,000	25,000	116.00%
Totals:	**$ 270,000**	**$ 270,000**	**700.00%**

```
                 MEAN             =       100.00% *
                                                  * MEASURES
                 MEDIAN           =       100.00% * OF
                                                  * ASSESSMENT
                 WTD. MEAN        =       100.00% * LEVEL
                                                  *
                 GEOMETRIC MEAN   =        99.36% *


(COD) COEFFICIENT OF DISPERSION  =          9.90% * MEASURES
                                                  * OF
(COV) COEFFICIENT OF VARIATION   =         12.17% * UNIFORMITY
```

In Example 10, the sales with prices identical to those in Example 9 have closer assessed values. Measurements of level are unchanged except for the geometric mean, which is now closer to the other measures. However, variation between ratios of assessment has been reduced and there will be much better taxpayer equity.

Determining Uniformity

Available procedures and statistics which enable uniformity to be calculated or visualized include:

1. Range
2. Frequency Distribution
3. Histogram
4. Coefficient of Dispersion (COD)
5. Standard Deviation
6. Coefficient of Variation (COV)
7. Price-related Differential (PRD)

Range

After ratios are computed and arrayed or put in order from lowest to highest (or the opposite), the range can be computed by finding the difference between the highest and lowest ratios. In the first "Level vs Uniformity" example (Example 9), the range was 120% while in Example 10, it was only 32%. Larger ranges generally indicate poorer uniformity, but the frequency of outlying (very low or high) ratios is much more important than mere presence.

The range is the same (190%) in both of the following samples:

Sale #	Ratio Sample A	Ratio Sample B
1	10%	10%
2	10%	100%
3	10%	100%
4	200%	100%
5	200%	100%
6	200%	200%

Uniformity obviously is better in Sample B. Results in sample A are more likely to be indicative of systematic appraisal error.

Frequency Distribution

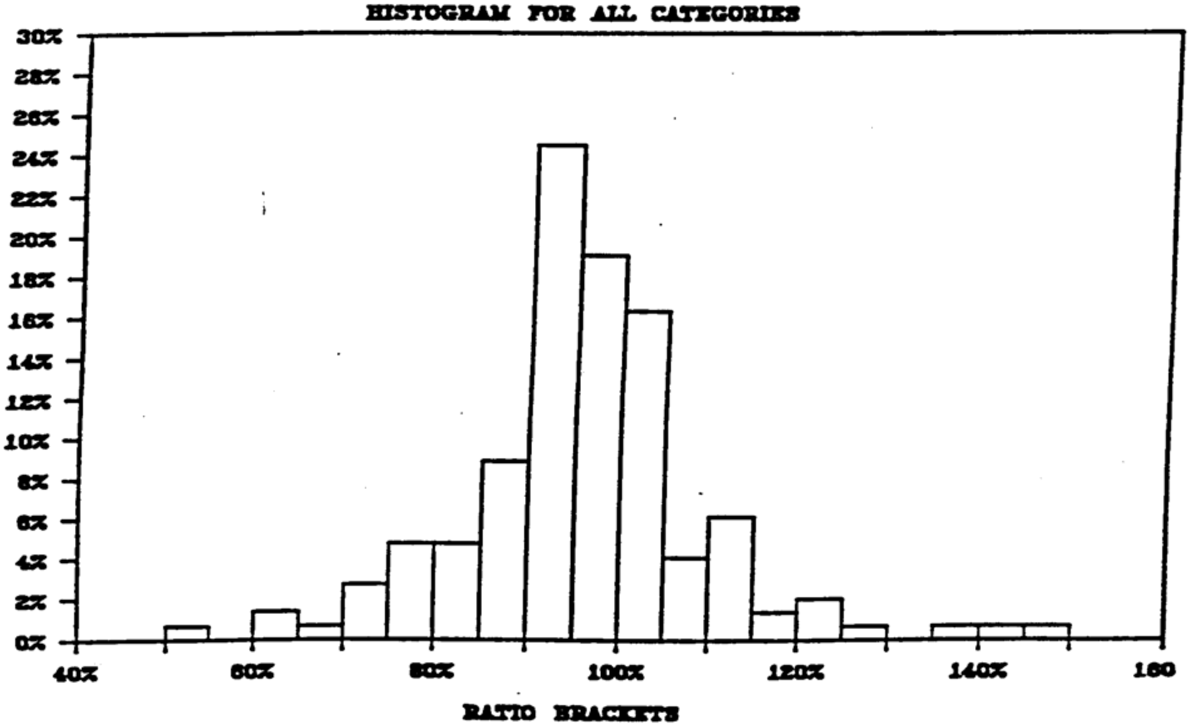
Arrayed ratios can be grouped into brackets typically 5% or 10% in width. A sample frequency distribution follows:

Frequency of Ratios of Assessed Value to Sales Price/Appraisal Value																						
	#	Ratio Brackts (%)																				
Category	in	0	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150
	Sample	54	59	64	69	74	79	84	89	94	99	104	109	114	119	124	129	134	139	144	149	Over
Resid w/Imp.	74	0	0	1	1	2	5	3	9	23	12	8	2	5	0	1	0	0	0	1	1	0
Resid. Land Only	0																					
Rural Inv.	20							2		5	4	7	1		1							
Commercial	12							1	1	3	3	2				1	1					
Impr. Rural Resid. Subd.	14			1			2				5	2	2	2								
Impr. Rural Resid. Tracts	14	1				1		1	1	2	1	5	1			1						
Unimpr. Rural Resid. Subd.	0																					
Unimpr. Rural Resid. Tracts	0																					
Mfg. Housing	13					1			2	3	3			2	1				1			
Totals:	147	1	0	2	1	4	7	7	13	36	28	24	6	9	2	3	1	0	1	1	1	0

Although this county had 1 improved residential ratio between 60% and 65% and 1 between 145% and 149%, twenty-three ratios in this category were between 90% and 95%. Forty-five ratios, or 61% of the sales in this category, fell between 90% and 110% of market value. The frequency distribution shows the concentration of ratios within certain brackets or intervals and gives an indication of the degree of uniformity.

Histogram

The histogram represents a picture of the frequency distribution. It is plotted by determining the percent of ratios in each frequency distribution bracket and can be drawn for individual categories or the entire county, as in this plot of the preceding frequency distribution:



Flat histograms or those without central peaks indicate poor uniformity, while the above histogram shows few outliers and good uniformity. By superimposing a histogram on the "normal" curve, the normality of ratio study data can often be ascertained.

Coefficient of Dispersion (COD)

The COD is a direct mathematical measurement of uniformity. It is based on how far each ratio differs from the median and is defined as the average percentage difference between each ratio and the median ratio. The COD is always expressed as a percent of the median and is computed using the following formula:

$$\mathrm{AAD} = \frac{\sum \left| \mathrm{A}_i / S_i - \widetilde{A/S} \right|}{n} \qquad \mathrm{COD} = \frac{100 * \mathrm{AAD}}{\widetilde{\mathrm{A/S}}}$$

Where: AAD is the average absolute deviation;

$\sum$ means "the sum of";

| | means absolute value disregarding ± sign;

A_i/S_i represents each individual ratio;

$\widetilde{A/S}$ is the median ratio.

COMPUTATION OF THE COD

Example 11:

Assessed Value	Sale Price	Ratio	Difference Between Each Ratio & Median
$ 21,000	$ 25,000	84.00%	16.00%
44,000	50,000	88.00%	12.00%
28,000	30,000	93.33%	6.67%
60,000	60,000	100.00%	0.00%
32,000	30,000	106.67%	6.67%
56,000	50,000	112.00%	12.00%
29,000	25,000	116.00%	16.00%
Total Difference:			**69.33%**

(AAD) AVERAGE DIFFERENCE = 9.90%

COD = 9.90%

In Example 11, the COD and the AAD were exactly the same. This will occur only if the median is 100%.

COMPUTATION OF THE COD

Example 12:

Assessed Value	**Sale Price**	**Ratio**	**Difference Between Each Ratio & Median**
$ 10,500	$ 25,000	42.00%	8.00%
22,000	50,000	44.00%	6.00%
14,000	30,000	46.67%	3.33%
30,000	60,000	50.00%	0.00%
16,000	30,000	53.33%	3.33%
28,000	50,000	56.00%	6.00%
14,500	25,000	58.00%	8.00%
		Total Difference:	**34.67%**

(AAD) AVERAGE DIFFERENCE = 4.95%

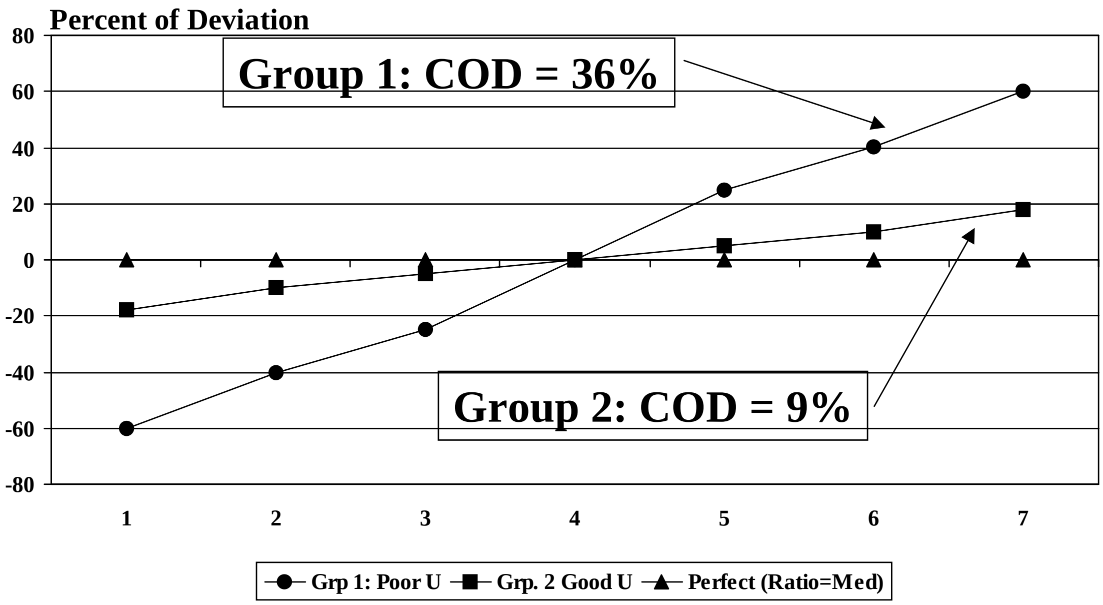
COD = 9.90%

Although the median is lower (50%) in Example 12, the relative spread between ratios is the same as in Example 11 and the COD is also the same.

This means that an average difference of 4.95% around a median of 50% represents the same degree of uniformity as an average difference of 9.9% around a median of 100%.

The following chart compares uniformity of two other samples, each consisting of seven sales:

Level vs. Uniformity
Deviation from Median



Compares 2 groups of 7 sales

If Deviation = 0, uniformity is perfect

Both groups of sales have exactly the same median and are therefore at the same level. Except for the one sale in each group that is sold for its assessed value (the median was 100% in both groups), all other sales prices deviated from assessments. The difference between the two groups is in the magnitude of the deviation, which was much greater in Group 1, with a COD of 36%, than in Group 2, with a COD of 9%.

To meet standards that have been established for uniformity, the COD must be 15% or less for improved residential property and 20% or less for unimproved property, manufactured housing and commercial property.

A practical grading system for improved property (add 5 points to this scale for unimproved property) would be:

COD (%)	**UNIFORMITY CONDITION**
< 5	Questionable
5-10	Excellent
10-15	Good
15-20	Somewhat Poor
20-30	Poor
> 30	Very Poor

Since typical marketplace variation precludes perfection in mass (or even single parcel) appraisal, CODs less than 5% are virtually impossible to obtain unless assessments are adjusted to sales prices on individual properties. Three exceptions, in which unusually low CODs may be expected, are:

1. Subdivisions in which lot price is strictly controlled by a developer;
2. Areas in which all improvements are identical or very similar (possibly condominiums);
3. Agricultural land, because of its non-market basis.

The COD is considered the best overall indicator of uniformity, since it is based on the median which is not distorted by high or low ratios. It does ***not*** however enable predictions concerning the proportion of property that is assessed within a particular range of the typical assessment level. For example, given a median of 90% and a COD of 10%, we would know that the typical property is assessed between 81% and 99% of market value (a COD of 10% and a median of 90% equates to a ±9 point range around the median ratio). However, we could not estimate the percentage of properties overvalued (100% +) or within any given range (i.e.: 90% - 110%).

Some prediction with the COD may be possible, since, in a normal distribution, the COD may be multiplied by 1.25 to approximate the COV.

Standard Deviation

The portion of property in the population having ratios within a particular range can be predicted from the standard deviation. This statistic is computed based on the unweighted mean ratio. The predictions that are made require the following assumptions:

1. The sample is representative of all property in the category being studied.
2. The sample has been selected randomly.
3. The ratios are normally distributed on either side (high or low) of the mean.

When these assumptions are known to be true the standard deviation, when computed, indicates the following:

Example 13: PREDICTING WITH THE STANDARD DEVIATION.

Given: The mean for all three groups is 95%.

Range of Ratios for indicated portion of property:				
Group	**Standard Deviation (%)**	**68% of prop.**	**95% of prop.**	**99% of prop.**
1	10	85 – 105%	75 – 115%	65 – 125%
2	20	75 – 115	55 – 135	35 – 155
3	30	65 – 125	35 – 155	5 - 185

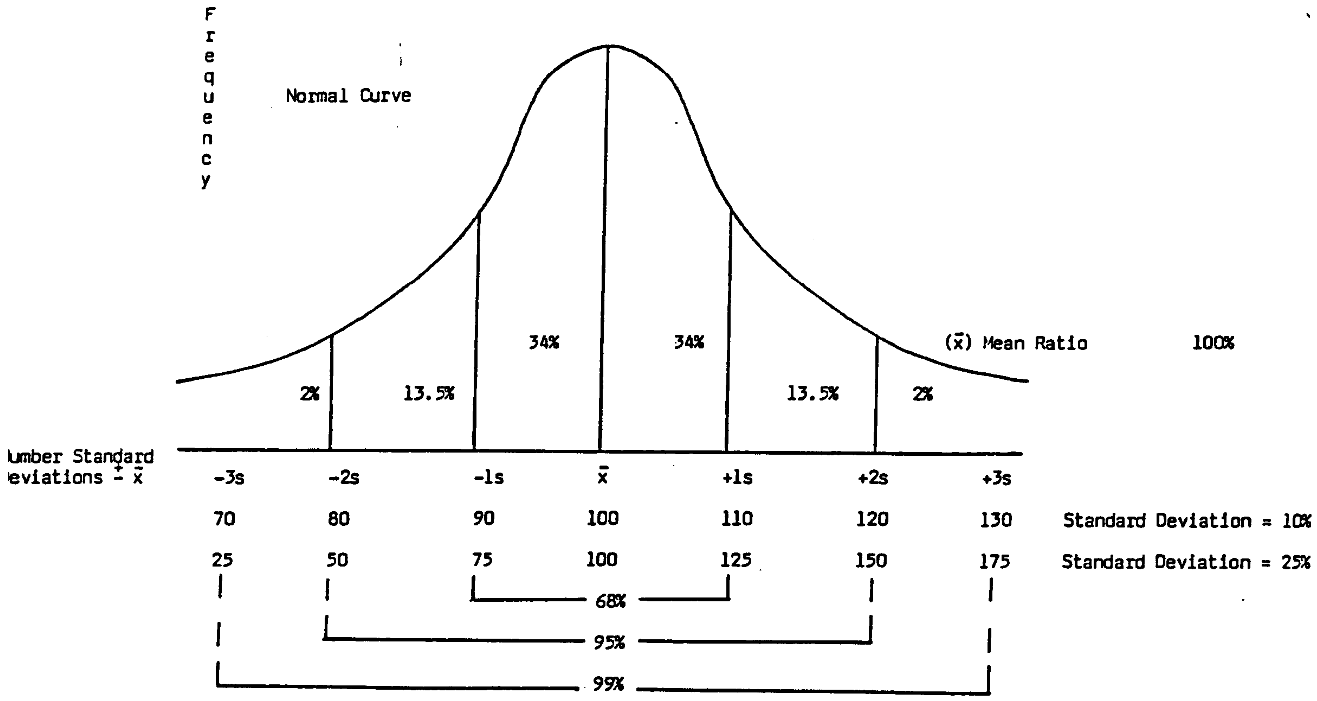
This distribution follows the rule:

# of standard deviation from mean	**% of property found within this range**
± 1	68%
± 2	95%
± 3	99%
± 4	Typically the entire range

In the above example, Group 1 has the best uniformity and can be predicted to have 99% of all property in the category studied assessed between 65% and 125% of market value. In Group 3, with poor uniformity but the same mean assessment level, only 68% of the property is in this range. The other 32% is assumed to be evenly split into higher and lower ratio groups. Therefore, 16% of the property in Group 3 is assessed more than 25% above market value, while 16% is at least 35% too low (below 65%). In Group 1 only 0.5%(1 property in 200) is assessed above 125% or below 65%. Another illustration of the proportions that can be predicted from the standard deviation is:

FIGURE 1

STANDARD DEVIATION - HYPOTHETICAL COUNTY



Since the means are 100% in the 2 categories shown in Figure 1, the ranges based on the standard deviations are centered around 100%. If the mean ratio is distorted by outliers, the predictive ability of the standard deviation is diminished. Even if the distribution of ratios is such that the bell-shaped curve illustrated above does ***not*** exist, the following predictions are still valid:

NON-NORMAL DISTRIBUTION EXAMPLE	
# of standard deviation from mean	**% of property found in this range**
± 1	Unknown
± 2	75%
± 3	89%

The Coefficient of Variation (COV) is an expression of the standard deviation as a percent of the mean. This "standardizes" the standard deviation so that the COV can be interpreted without additionally knowing the mean. In other words, the COV has exactly the same meaning for a sample with a mean ratio of 40% as for a sample with a mean ratio of 120%.

The standard deviation and COV are computed using the following formulas:

$$s = \sqrt{\frac{\sum (A_i / S_i - (\overline{A/S})^2}{(n-1)}} \qquad COV = \frac{100 * \mathrm{s}}{\overline{\mathrm{A/S}}}$$

where: s is the standard deviation;

$\sum$ means "the sum of";

n is the number of sales in the sample;

A_i/S_i represents each individual ratio;

$\overline{A/S}$ is the mean ratio.

COMPUTATION OF THE STANDARD DEVIATION & THE COV

Example 14:

Assessed Value ($)	Sale Price ($)	Ratio	Difference Between Each Ratio & Mean	Difference Squared
10,000	25,000	40.00%	-60.00%	3600%
30,000	50,000	60.00%	-40.00%	1600%
22,500	30,000	75.00%	-25.00%	625%
60,000	60,000	100.00%	0.00%	0%
37,500	30,000	125.00%	25.00%	625%
70,000	50,000	140.00%	40.00%	1600%
40,000	25,000	160.00%	60.00%	3600%
Sum of Ratios:		**700.00%**	**Sum of Squares:**	**11650%**

MEAN RATIO: 100.00% Sum of squares divided by sample size –1: 1942%
Square Root: 44.06%

THE STANDARD DEVIATION IS 44.06%

THE COV (COEFFICIENT OF VARIATION) IS
THE STANDARD DEVIATION DIVIDED BY THE MEAN: 44.06%

COMPUTATION OF THE STANDARD DEVIATION & THE COV

Example 15:

Assessed Value ($)	**Sale Price ($)**	**Ratio**	**Difference Between Each Ratio & Mean**	**Difference Squared**
21,000	25,000	84.00%	-16.00%	256%
44,000	50,000	88.00%	-12.00%	144%
28,000	30,000	93.33%	-6.67%	44%
60,000	60,000	100.00%	0.00%	0%
32,000	30,000	106.67%	6.67%	44%
56,000	50,000	112.00%	12.00%	144%
29,000	25,000	116.00%	16.00%	256%
Sum of Ratios:		**700.00%**	**Sum of Squares:**	**889%**

MEAN RATIO: 100.00%
SUM OF SQUARES DIVIDED BY
SAMPLE SIZE - 1: 148%
SQUARE ROOT: 12.17%

THE STANDARD DEVIATION IS 12.17%

THE COV (COEFFICIENT OF VARIATION) IS
THE STANDARD DEVIATION DIVIDED BY THE MEAN: 12.17%

In these two examples the following predictions can be made:

Range of Ratios for indicated portion of property:			
Example	**Standard Deviation**	**68% of property**	**95% of property**
14	44.06%	55.94 – 144.06	11.88 – 188.12
15	12.17%	87.83 – 112.17	75.66 – 124.34

Obviously, uniformity is much better in Example 15, where only 5% of all property is predicted to have ratios outside of the range from 75.66% to 124.34% of market value.

The standard deviation is dependent on the mean. A lower mean ratio will result in a lower standard deviation, often giving false indication of better uniformity. To more accurately judge uniformity regardless of the assessment level, the Coefficient of Variation (COV) must be determined as in the following example:

COMPUTATION OF THE STANDARD DEVIATION & THE COV

Example 15a: STANDARD DEVIATION VS. COV

Assessed Value ($)	Sale Price ($)	Ratio	Difference Between Each Ratio & Mean	Difference Squared
10,500	25,000	42.00%	-8.00%	64%
22,000	50,000	44.00%	-6.00%	36%
14,000	30,000	46.67%	-3.33%	11%
30,000	60,000	50.00%	0.00%	0%
16,000	30,000	53.33%	3.33%	11%
28,000	50,000	56.00%	6.00%	36%
14,500	25,000	58.00%	8.00%	64%
Sum of Ratios:		**350.00%**	**Sum of Squares:**	**222%**

MEAN RATIO: 50.00% SUM OF SQUARES DIVIDED BY SAMPLE
SIZE - 1: 37%
SQUARE ROOT: 6.09%
STANDARD DEVIATION IS 6.09%

THE COV (COEFFICIENT OF VARIATION) IS
THE STANDARD DEVIATION DIVIDED BY THE MEAN: 12.17%

If Examples 15 and 15a are compared, the standard deviation in 15a is seen to be 1/2 of the standard deviation in Example 15. The COVs however, are identical. The lower standard deviation matches the lower mean (50% vs 100%) in Example 15a. Uniformity relative to the mean is considered identical in these 2 examples.

To meet standards for uniformity the COV (not the standard deviation) must be 20% or less for improved residential property, and 25% or less for unimproved property, commercial property, and manufactured housing.

A practical grading system (add 5 points for unimproved property) would be:

COV (%)	**Uniformity Condition**
< 5	Questionable
5-10	Excellent
10-20	Good
20-30	Somewhat poor
30-40	Poor
> 40	Very poor

Price-related Differential (PRD)

Property appraisals can sometimes tend to place unequal tax burdens on either high or low value property. Inequity of this type is termed **vertical**, meaning that properties in different value strata are assessed differently in comparison to market value. Assessments would be considered progressive if higher priced property were to be over-assessed in relation to lower priced property. This would occur, for instance, if most $100,000 value properties were appraised at $90,000 (90%) while $30,000 properties were appraised at $24,000 (80%). The opposite situation would be considered regressive.

Manufactured housing is often regressively treated, with older, smaller, lower value properties typically assessed at or over market value, while larger new properties often are under market value.

Bias in favor of high or low priced properties is measured with an index statistic known as the Price-related Differential (PRD). This statistic is computed using the following procedure:

$$\text{PRD} = \frac{\overline{A/S}}{\overline{A}/\overline{S}}$$

where, $\overline{A/S}$ is the mean ratio;

$\overline{A}/\overline{S}$ is the weighted mean ratio.

<table>
<tr><th colspan="4">Three types of results can be demonstrated:</th></tr>
<tr><th>Price-Related Index</th><th>Meaning</th><th>Favors</th><th>Type of Bias</th></tr>
<tr><td>1.00</td><td>Low and High priced property treated same.</td><td>Neither</td><td>None</td></tr>
<tr><td>< 1.00</td><td>Lower ratios on high priced property</td><td>High Priced</td><td>Regressive</td></tr>
<tr><td>> 1.00</td><td>Lower ratios on low priced property</td><td>Low Priced</td><td>Progressive</td></tr>
<tr><td colspan="4">Standard: 0.98 – 1.03 = OK</td></tr>
</table>

If the PRD is between 0.98 and 1.03, the degree of bias or vertical tax inequity is not considered significant. However, the Mann-Whitney test described following this section is considered more definitive and should be used in addition to the PRD, particularly for large samples (more than 100 sales).

The PRD is computed by dividing the mean by the weighted mean. This calculation effectively measures the distortion in the weighted mean caused by high or low ratios on high or low valued property. Since the mean is not affected by value, but only by ratios, this measurement can serve as a baseline for the comparison. The following examples show the computation of the PRD and demonstrate the tax inequities represented:

PRD COMPUTATION CHART
EXAMPLES OF VARIOUS CONDITIONS:

Example A:

Sale #	Assessed Value	Sale Price	Ratio
1	$ 25,000	$ 20,000	125.00%
2	24,000	30,000	80.00%
3	31,000	40,000	77.50%
4	40,000	50,000	80.00%
5	60,000	60,000	100.00%
6	79,000	70,000	112.86%
Totals:	**259,000**	**270,000**	**575.36%**

WEIGHTED MEAN = 95.93%
MEAN = 95.89%
PRD = 1.00*
*DOES NOT FAVOR LOW OR HIGH PRICED

Example B:

Sale #	Assessed Value	Sale Price	Ratio
1	$ 30,000	$ 20,000	150.00%
2	40,000	30,000	133.33%
3	45,000	40,000	112.50%
4	50,000	50,000	100.00%
5	40,000	60,000	66.67%
6	45,000	70,000	64.29%
Totals:	**250,000**	**270,000**	**626.79%**

WEIGHTED MEAN = 92.59%
MEAN = 104.46%
PRD = 1.13**
**FAVORS HIGH PRICED

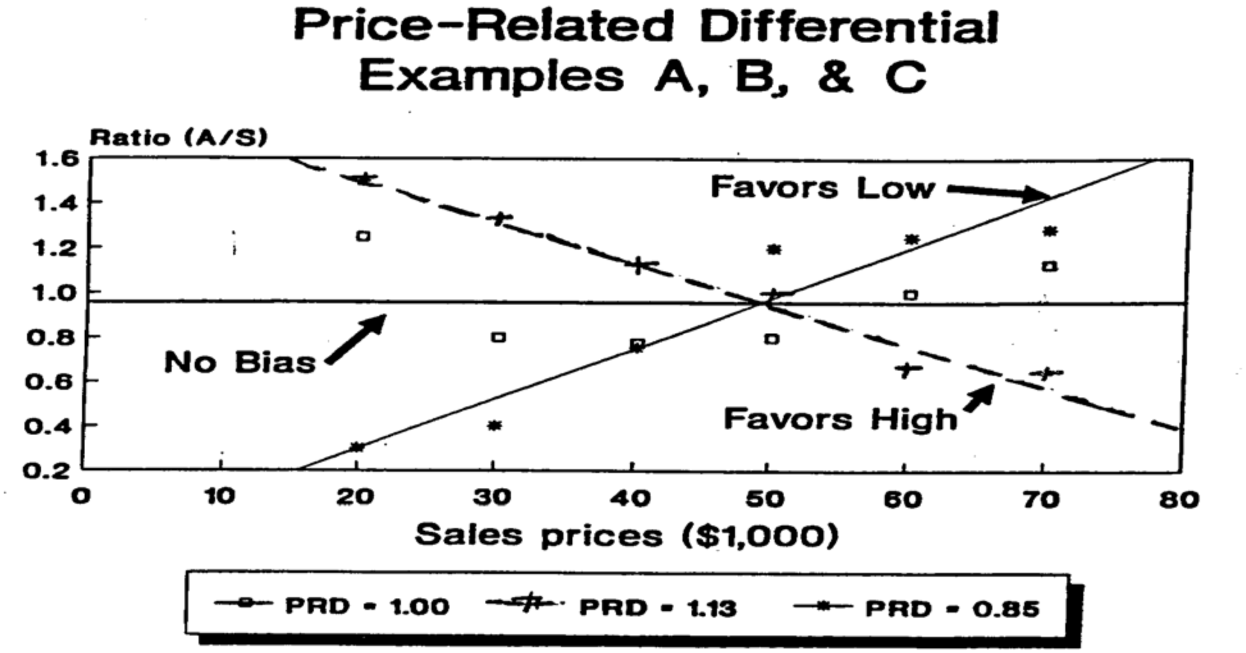
PRD COMPUTATION CHART
EXAMPLES OF VARIOUS CONDITIONS:
Example C:

Sale #	Assessed Value	Sale Price	Ratio
1	$ 6,000	$ 20,000	30.00%
2	12,000	30,000	40.00%
3	30,000	40,000	75.50%
4	60,000	50,000	120.00%
5	75,000	60,000	125.00%
6	90,000	70,000	128.57%
Totals:	**273,000**	**270,000**	**518.57%**

WEIGHTED MEAN = 101.11%
MEAN = 86.43%
PRD = 0.85***
***FAVORS LOW PRICED

These examples use the same group of sales and show the effects of different assessments. Although no group is assessed perfectly, there is no discernable distortion based on value in group A. In group B, however, assessment ratios clearly decline as value (sale price) increases; the assessments favor higher price property. The opposite occurs in group C.

Graphically, the 3 examples appear as follows:



Mann-Whitney Test

As a measure of the degree of value related equity problems, the PRD has certain drawbacks. In small samples, the PRD is very sensitive to distortion caused by the presence of a very small number of “outlier” type sales. One or two high value, low ratio sales (or vice versa) can easily result in a PRD which appears to indicate a significant value related problems. However, the significance of these results may not be provable in a statistical sense. When this is the case, value related inequity may not be occurring even though the PRD does not meet standard. Similarly, in very large samples, the PRD may be within acceptable standards, yet there may still be value related appraisal problems occurring in a small sector of the properties. Perhaps, for example, appraisals are low on homes over $500,000, but this group constitutes only 15 sales in a sample of 800. If vertical equity has been achieved in all other strata, it is unlikely that the PRD will indicate any problem.

To guard against inaccurate judgments and decisions based on the PRD, ratio studies can be developed to test specific values strata. In addition, the significance of value related inequity can be tested using statistical tests, such as the Mann-Whitney test, which can be used to compare the rank of the ratio corresponding to each sale to each sale price. If there is no value related inequity, these ranks will be relatively randomly dispersed. If most of these high-priced sales have low or high ranks, value related inequity is indicated.

The Mann-Whitney test is calculated by finding *U* from the following formula and then testing the statistical significance with a *z* score.

$$U = n_1 n_2 + \frac{n_1(n_1+1)}{2} - \mathrm{R}_1$$

Where: n_1 is the number of sales in the group < the mean sale price,
n_1 is the number of sales in the group > the mean sale price
R_1 is the sum of the ranks in the group < the mean price.

Once *U* is calculated, the *z* score is determined using the following formula:

$$z = \frac{U - (n_1 n_2)/2}{\sqrt{(n_1 n_2)(n_1 + \mathrm{n}_2 + 1)/12}}$$

If *z* is greater than 1.96 or less than –1.96, there is a statistically significant difference between the ratios in the two groups and value related inequity is likely. The Mann-Whitney test is demonstrated in Appendix III g. For this test to be used, the smaller group must have no fewer than 8 ratios. (See **PAAA**, IAAO, 1990 for additional restrictions.)

STATISTICAL MEASURES OF RELIABILITY

All of the statistics previously discussed represent measurements made on sample data. In fact, by definition, statistics always concern samples. The ultimate purpose of all of the measurements, however, is to determine assessment conditions for the entire group or population of properties in each category studied.

Measurements made directly on populations provide parameters or facts. Since we must indirectly measure the population from a limited sample, based only on properties that sell, we are forced to estimate the population parameters. The precision and validity of this estimation is based on several factors including:

1. **Sample Randomness:** Sample acquisition should be unbiased with every property having an equal chance for selection. Although there is no direct bias in the way we choose sales to be used, each property probably does not have an equal opportunity to sell and get into the ratio study. This requirement should be considered partially met by our sampling procedure.

2. **Sample Representativeness:** This requires that the sample be drawn from the population under investigation and that individual observations (types of properties) occur in the sample in approximately the same frequency as in the population. Stratification by area and category of property helps fulfill this requirement. However, there may be a tendency in some unsegregated areas for extensive sales activity in one subdivision and few or no sales in another. If the property characteristics and other market influences are similar in the two areas, there is no problem. However, if the areas' economic forces differ significantly, the requirement for representativeness may not be fully satisfied.

3. **Normality:** This is the requirement for assessment ratios to be randomly distributed with respect to the mean throughout both the sample and population. Many sources consider this doubtful in regard to assessment ratios. Usually, however, only large ratio study samples can be proven not normal with any significant degree of certainty. Non-parametric statistics may be employed to avoid inaccurate parameter estimates that may otherwise result in non-normal situations.

Measuring Ratio Study Reliability

The reliability of ratio study results is the most important single aspect of the ratio study. Decisions made using ratio studies have the potential of affecting hundreds of thousands of taxpayers across the state. If the results used as a basis for these decisions are not reliable, two types of errors can occur:

1. Category values may be adjusted (up or down) when, in fact, no adjustment is warranted.
2. Category values may be considered satisfactory when, in fact, adjustments should be made.

These errors are equally serious in terms of taxpayer inequity and may be made by either the state though equalization decisions or the county through appraisal decisions.

Given a reasonably random and representative sample, reliability depends on two factors:

1. Sample size
2. Sample uniformity

Large, uniform samples produce more reliable results than small samples with poor uniformity.

Ratio study reliability can be measured using two different approaches:

1. Direct measurement of probability of true mean between 90% and 110% (or any selected level);
2. Indirect measurement of range within which true mean or median is likely to be found.

The second of these approaches involves the development of 95% (or other appropriately significant) confidence intervals based on the:

1. Mean,
2. Median, or
3. Weighted mean.

Regardless of the base statistic (mean, median, weighted mean), the confidence interval determined in a ratio study indicates the range within which we are 95% (or any other selected degree of certainty) certain that the true assessment level occurs.

In other words, a confidence interval of 85% - 115% indicates that we are 95% confident that if all property in the category being studied were to be sold and assessment ratios computed, the true overall average level of assessment would be between 85% and 115%.

The following chart illustrates the effects of sample size and uniformity (COV) on the mean based confidence interval (the mean is assumed to be 100% in each sample shown):

Sample Size	**Coefficient of Variation (COV) (%)**	**95% Confidence Interval (CI)**
5	10	87.6 – 112.4
10	10	92.8 – 107.2
50	10	97.2 – 102.8
100	10	98.0 – 102.0
5	20	75.2 – 124.8
10	20	85.7 – 114.3
50	20	94.5 – 105.5
100	20	96.1 – 103.9
5	30	62.8 – 137.2
10	30	78.5 – 121.5
50	30	91.7 – 108.3
100	30	94.1 – 105.9

Note: Confidence intervals developed for determining compliance with assessment level standards will be slightly narrower, because 90% (or, in some cases, 80%) intervals, rather than 95% intervals, are being determined. Also, these intervals are based on the median and therefore will only coincidentally be symmetrical.

Computation of Probability

Ratio study probabilities are computed to determine the chance that sale of all properties in a category within a jurisdiction would indicate a particular mean level of assessment. For ratio study standards, a desirable range for the true level of assessment is between 90% and 110% of market value. We therefore calculate the probability that this level has been attained. This probability is calculated using:

1. the "t" test for samples of 30 or fewer sales;
2. the "z" test for larger samples.

The formula for z or t is as follows:

$$\text{z or t} = \frac{\overline{\text{A/S}} - \mu}{\text{s} / \sqrt{n}}$$

where: $\overline{A/S}$ is the sample mean ratio;
μ ("mu") is the population mean to be tested;
s is the sample standard deviation;
n is the sample size.

Once this equation is solved, standard tables (Appendices IV and V) must be consulted to determine the probability corresponding to the computed t or z value. The following examples demonstrate this procedure:

Example 16:

A sample of 36 sales has a mean ratio of 85% and a standard deviation of 10%. We wish to determine with a 5% maximum error the probability that the true mean level of assessment is between 90% and 110% of market value.

Since this probability question requires determining the probability that the true mean is within a given range, two separate calculations must be done:

1. Find the probability that μ >90% by: $z = \frac{.85 - .90}{.10/\sqrt{36}} = -3.0$

2. Find the probability that μ >110% by: $z = \frac{.85 - 1.10}{.10/\sqrt{36}} = -15.0$

Looking up -3.0 in the z table in Appendix IV indicates that there is a 0.13% probability that the true mean (μ) exceeds 90%. Looking up -15 indicates that there is virtually no probability that the true mean exceeds 110%. Therefore, the probability that the true mean is between 90% and 110% must be 0.13%. If we decide that assessment level is unacceptably low and values should be increased, there will be a 0.13% probability that the mean level was already acceptable.

Example 17:

A sample of 81 sales has a mean ratio of 87% and a standard deviation of 18%. We wish to determine the probability of a true mean between 90% and 110%.

1. the z score for μ > 90%: $z = \frac{.87 - .90}{.18/\sqrt{81}} = -1.50$

2. the z score for μ > 110%: $z = \frac{.87 - 1.10}{.18/\sqrt{81}} = -11.50$

From the z table, the probability that the true mean is greater than 90% is 6.68%. There is virtually no probability that the true mean is greater than 110%. Therefore, the probability that the true mean is between 90% and 110% is 6.68%.
Probabilities cannot exceed 100% or absolute certainty. However, the standard in use in Idaho requires 5% or higher probability that the

true mean is in the 90% to 110% range, for samples using the mean to determine compliance. This means that, for such categories, county determined assessed values will be considered market value provided that there is at least a 5% chance that an overall 90% to 110% range has been attained and provided the sample mean has not been outside of the 90% to 110% range previously (see Standards).

With reference to examples 16 and 17, assessment level would be considered satisfactory in Example 17, but unsatisfactory in Example 16 (See: Standards and Equalization section).

Computing Confidence Intervals

The Mean Confidence Interval

The following formula can be used to compute this confidence interval:

$$CI(\overline{A/S}) = \overline{A/S} \pm \left[\frac{(t)*(s)}{\sqrt{n}}\right]$$

Where: CI $(\overline{A/S})$ is the confidence interval around the mean;

t is the constant from the appropriate column of the "t table" (Appendix V) based on n-1 degrees of freedom; (Note: the column to be used depends on the selected probability of the mean being outside of the interval - to be 95% sure the mean is within the interval, select the .05 probability column.)

s is the sample standard deviation;

n is the sample size.

In calculating 95% confidence intervals using the means and other information in examples 16 and 17, we find:

Example 16:

Mean = 85%, Standard deviation = 10%, sample size = 36.

$$95\%\ \text{CI} = .85 \pm \left[\frac{(1.96)*(.10)}{\sqrt{36}}\right]$$

= .85 ± .03
= (.82 - .88)

We can be 95% confident that the true mean level of assessment is between 82% and 88% of market value.

Example 17:

Mean = 87%, standard deviation = 18%, sample size = 81.

$$95\%\ \mathrm{CI} = .87 \pm \left[\frac{(1.96)*(.18)}{\sqrt{81}}\right]$$

$= .87 \pm .04$
$= (.83 - .91)$

We can be 95% confident that the true mean level of assessment is between 83% and 91% of market value. For compliance testing, 1-tailed 95% intervals must be computed. These would use constants from the 0.1 probability column in Appendix V. For Example 17 this would mean substituting 1.645 for the 1.96 constant. This would change the confidence interval to .87 ±.03 or .84 - .90, which would barely meet standard. Except when noted for compliance testing purposes, confidence intervals shown in this manual are based on two-tailed computation methods. These intervals indicate a range within which, with 95% certainty, the true mean will lie.

In general, narrower confidence intervals indicate greater reliability and occur when large samples with good uniformity are available. The effects of sample size and uniformity on unweighted mean based confidence intervals is shown in the following examples:

CONFIDENCE INTERVALS Ratio Study Samples for Categories A through D				
Statistics	**A**	**B**	**C**	**D**
Mean	95	95	95	95
Standard Deviation	10	50	50	16
Sample Size	100	100	16	4
95% Confidence Interval (mean based)				
*UCL	96.96	104.8	121.6	120.5
*LCL	93.04	85.2	68.4	69.5
Width	3.92	19.6	53.2	51.0

In these 4 examples "UCL" indicates the upper confidence limit while "LCL" shows the lower limit of the interval. The width is the difference between these upper and lower limits. Sample A is the most reliable while sample C is the least reliable.

The Weighted Mean Based Confidence Interval

This interval provides information similar to that given by the unweighted mean based interval. Only the central point and calculation process have been changed. This interval is important when measuring reliability in any sample having a PRD significantly greater or less than 1.00.

The 95% confidence interval using the weighted mean can be found from the following formula:

$$95\%\ \mathrm{CI}\left(\overline{\mathrm{A}}/\overline{\mathrm{S}}\right) = \overline{\mathrm{A}}/\overline{\mathrm{S}} \pm t_{\propto\ 05} * (\mathrm{S}[\overline{\mathrm{A}}/\overline{\mathrm{S}}]),$$

where: $\bar{A}/\bar{S}$ is the weighted mean,

$t_{\propto\ 05}$ is a constant from the .05 error (probability) column of the t-table using n-1 degrees of freedom,

$S[\overline{\mathrm{A}}/\overline{\mathrm{S}}]$ is the standard error of the weighted mean.

The formula for is: $S[\overline{\mathrm{A}}/\overline{\mathrm{S}}]$

$$S\left[\bar{A}/\bar{S}\right] = \frac{\sqrt{\sum \overline{A^2} \ - 2\ (\bar{A}/\bar{S}\sum(A*S) + (\bar{A}/\bar{S})^2(\sum S^2)}}{\bar{S}\sqrt{(n)\ (n-1)}}$$

In the above formula, terms are used as follows:

A is assessed value;
S is sale price;
$\bar{S}$ is average sale price.

The following example demonstrates the use of this formula:

Example 18: Weighted Mean Confidence Intervals
Note: all values are expressed in thousands of dollars.

Sale	Assessed Value (A)	(A^2)	Sale Price (S)	(S^2)	(A) * (S)
1	10	100	40	1,600	400
2	20	400	30	900	600
3	30	900	30	900	900
4	30	900	25	625	750
5	25	625	19	361	475
6	20	400	12	144	240
Totals:	**135**	**3,325**	**156**	**4,530**	**3,365**

The mean ratio is 101.7% while the weighted mean is 86.54%.
The PRD is 1.17. Terms to be substituted into the formula are the following:

$n = 6$

$\overline{S}$ = 156/6 = 26 (average sales price)

$2(\overline{A}/\overline{S})\sum(A*S)$ = 2(0.8654)(3365) = 5,824.142

$(\overline{A}/\overline{S})^2(\sum S^2)$ = $(0.8654)^2(4530)$ = 3,392.595

With 5 degrees of freedom the t constant = 2.571 (Appendix V)
Substituting and solving the equation gives the following:

$$95\%\mathrm{CI(A/S)} = 0.8654 + 2.571 * \frac{\sqrt{3325 - 5824.142 + 3392.595}}{26 * \sqrt{6 * 5}}$$

$$= 0.8654 \pm 2.571 * \left\{\frac{29.89}{142.41}\right\}$$

$$= 0.8654 \pm 0.5396 = 0.3258 - 1.4050$$

The mean based interval for this example is 49.00% - 154.40%, substantially different from the weighted mean based interval above.

The Median Confidence Interval:

Reliability of results for samples or populations which are not normally distributed can best be tested by developing a median based confidence interval. This statistic is determined by a process whereby certain ratios are selected and represent lower or upper bounds on the interval. Outlying low or high ratios do not affect this selection process, which is based strictly on the number of ratios (sales) available in the sample. The procedure is taken from the IAAO, **PAAA** textbook and follows:

Median Confidence Interval Formula

Depending on even or odd count, one of two formulas is used to calculate the rank of the ratios corresponding to the upper and lower confidence interval limits.

If "n" (sample size) is **even:**

$$j = \left[\frac{1.96 * \sqrt{n}}{2} + 0.5 \right]$$

If "n" is odd:

$$j = \frac{1.96 * \sqrt{n}}{2}$$

where, j is the number of ratios to be counted up and down from the median to determine the rank of the upper and lower confidence interval. Before counting, the result (j) must always be rounded upward to the next integer.

Odd Example 19:

If n = 25, which is odd,

$j = \frac{1.96 * \sqrt{25}}{2} = 4.9$, which is rounded to 5

Since the median is the 13th ratio, the lower limit of the median confidence interval would be the 8th ratio (13-5) and the upper limit would be the 18th ratio (13+5).

Even Example 20:

If n = 16, which is even,

$$J = \frac{1.96 * \sqrt{16}}{2} + 0.5 = 4.42$$, which is rounded to 5

Since the median is between the 8th and 9th ratios, we count **down** 5 from the ratio above the median and **up** 5 from the ratio below the median. This gives ranks of 9 - 5, or 4 for the lower limit, and 8 + 5, or 13 for the upper limit.

Because of errors which may occur if the lowest and highest ratios are determined to be one or both of the confidence interval limits, this test is only considered valid for sample sizes of 9 or more. However, binomially based methods have been developed and published and allow 90% median confidence intervals to be computed more accurately for smaller samples. These procedures were incorporated into state conducted ratio studies beginning in 2007.

DETERMINING NORMALITY

Reliability testing methods based on the unweighted mean ratio depend on normal distribution of the sample. How closely a given sample distribution fits a normal distribution can be determined using various procedures as listed below:

Method	**Sample Size For Use**
Chi - Square (χ^2)	100 or greater
Binomial Approximation	25 - 99
Direct Binomial Test	Less than 25

The Chi Square Procedure

For this procedure, it is necessary to create a frequency distribution for the sample using at least 6 brackets or intervals. An expected frequency for the number of ratios that should fall within each of the chosen brackets can then be developed, assuming a perfectly normal distribution. Brackets should be chosen so that the expected frequency is at least 5 ratios in each bracket. For every 5 brackets, it is permissible for 1 to have an expected frequency of fewer than 5. If the expected frequency is much smaller (near 0), the presence of even a single ratio actually falling in the bracket will be enough to conclude that there is a non normal distribution. Therefore lower and upper end brackets should be combined and widened to prevent this distortion.

After expected frequencies have been computed and brackets finalized, observed (actual) frequencies and expected (hypothetical) frequencies are compared and the probability that the sample distribution is normal can be determined. Example 21 demonstrates this statistic:

Example 21:

sample size	$n = 114$
sample average ratio	$\overline{A/S} = 100.58$
sample standard deviation	$s = 15.22$

Bracket K

Equal to or			Expected Frequency (E)	Observed Frequency (O)
Greater than	but	Less than		
0%		80%	10.04	9
80%		90%	17.50	9
90%		100%	27.51	33
100%		110%	28.18	38
110%		120%	19.04	15
120%		130%	8.47	6
130%			3.05	4

Observed frequencies were taken directly from the ratio in the sample of 114 sales. Expected frequencies were calculated (see z test calculations following the χ^2 calculation). Chi-square is calculated as follows:

$$X^2 = \frac{\sum (\mathrm{O-E})^2}{E} =$$

$$\frac{(9-10.04)^2}{10.04} + \frac{(9-17.50)^2}{17.50} + \frac{(33-27.51)^2}{27.51}$$

$$+\frac{(38-28.18)^2}{28.18} + \frac{(15-19.04)^2}{19.04} + \frac{(6-8.47)^2}{8.47} + \frac{(4-3.05)^2}{3.05} = 10.63$$

(χ^2 here is pronounced "chi-squared" and $\sum$ means "the sum of" and is pronounced "sigma".)

Expected frequencies are determined by using the z test and finding the probable number of ratios in a given bracket assuming normality. For example, for the 80% - 90% bracket, the following calculation is used:

Probability of A/S (any ratio) less than 80%:

$$z = \frac{A/S - \overline{A/S}}{s} = \frac{80 - 100.58}{15.22} = -1.35$$

The probability that any ratio is less than 80% is determined from the z table and here is 8.85%.

$$z = \frac{\mathrm{A/S} - \overline{\mathrm{A/S}}}{\mathrm{s}} = \frac{90\ -\ 100.58}{15.22} = -0.70$$

The probability that A/S is less than 90% is 24.20%.

The probability of a given ratio between 80% and 90% is therefore 24.20% - 8.85%, which equals 15.35%. This percent (15.35%) multiplied by 114 ratios (n) results in an expected 17.5 ratios in the specified bracket, assuming a normal distribution.

Normality is always the assumed state, unless we have sufficient evidence to prove that a distribution is non-normal. From the chi-square table (Appendix VI), we can find that to reject our hypothesis of a normal distribution with 95% confidence, the calculation would have to result in a chi-square statistic of 12.59 (There are 7 brackets or 6 degrees of freedom). Since our answer was 10.63 and this does not exceed the critical value of 12.59, we must conclude that the distribution of ratios in our sample is normal.

In addition to allowing us to determine normality, the chi-square calculation process shows clearly the ratio brackets within which the actual results (observed frequencies) differ the most from the expected, "normal" results. In the sample used in Example 21 the greatest differences occurred in the 80% - 90% bracket. The deviation in this bracket accounts for nearly one-half (4.13) of the total chi-square value (10.63) as follows:

$X^2 = \frac{(\mathrm{O} - \mathrm{E})^2}{E}$ (No $\sum$, since this is for one bracket)

For the 80% to 90% bracket, E = 17.50 and O = 9, therefore,

$$X^2 = \frac{(17.50 - 9)^2}{17.50} = 4.13$$

Non-normal determinations are often caused by an absence of ratios that are very low or very high. It is a common misconception that ratios which deviate significantly from the average occur only in non-market value sales. This error may result in exclusion of many sales which might enhance reliability and permit a conclusion of normality. The following chart demonstrates typical ratio ranges that would be anticipated in normally distributed samples.

Typical Large Sample (n >100) Ratio Range, Mean $(\overline{A/S})$ = 100

Uniformity (Comments)	Standard Deviation	Expected Range (%)
Excellent	8	68 - 132
Very Good	12	52 - 148
Good	15	40 – 160
Borderline	20	20 – 180

Note: the above chart reflects the principle that the range is ordinarily approximated by the mean ±4 standard deviations. For small samples, this is reduced to ±3 standard deviations:

Typical Small Sample Range (Mean = 100%)	
Standard Deviation (%)	**Expected Range**
8	76 – 124
12	64 – 136
15	55 – 145
20	40 – 160
30	10 - 190

Binomial testing procedures are more appropriate for samples with less than 100 ratios; results are similar to those in Example 21.

Binomial Test of Normality

$$z = \frac{0.5(n-1) - x_s}{\sqrt{0.25n}}$$

Samples composed of between 25 and 99 ratios may be tested for normality by using a binomial approximation method employing the following formula:

Where: n is the total number of ratios;
x_S is the number of ratios in the smaller of the two groups:
1. Number of ratios greater than the mean, or
2. Number of ratios less than the mean.
z is the score to be tested against a critical value to determine normality.

Example 22:

Sample size (n) = 25

Mean $\left(\overline{A/S}\right)$ = 100%

Standard Deviation (s) = 22.8%

Array of Ratios:

Sale #	Ratio (%)	Sale #	Ratio (%)
1	80%	14	93
2	82	15	93
3	83	16	95
4	85	17	97
5	85	18	98
6	86	19	99
7	86	20	101
8	86	21	109
9	87	22	137
10	89	23	145
11	90	24	150
12	92	25	159
13	93		

Number of ratios exceeding the mean: 6
Number of ratios less than the mean: 19

Using the data from the preceding example, where x_s would be 6 (the number of ratios in the smaller group) and n equals 25, we would calculate:

$$Z = \frac{0.5(25-1)-6}{\sqrt{.25(25)}} = 2.4$$

The critical value of z necessary to reject the hypothesized normality at the 95% confidence level is 1.96 (a constant used for this purpose).

Since our answer was greater than 1.96, we must conclude that the sample is ***not*** normally distributed. Since the standard deviation was 22.5% in Example 22, we would have expected a range of ± 67.5 points (3 standard deviations) corresponding to ratios from 32% to 168%. Since we are much closer to meeting the upper limit of this range, it appears very likely that sales with even moderately low ratios may have been deleted from the sample.

In non-normal distributions, the median based statistics are usually the best indicators of assessment conditions. The median is 93% in Example 22 and probably gives a truer picture of level than the mean in this case.

Direct determination of normality

For samples with less than 25 ratios, probability statements concerning normality can be derived directly from binomial tables. The following table shows the maximum number of ratios that may occur in the smaller of the group of ratios, greater or less than the mean, and still permit us to conclude that we have a normal distribution.

Binomial Table	
Sample Size (n)	**Critical Value: Normal if number of ratios in small group is at least:**
5 or less	N/A
6	1
7	1
8	2
9	2
10	2
11	3
12	3
13	4
14	4
15	4
16	5
17	5
18	6
19	6
20	6
21	7
22	7
23	8
24	8
25	8

> The critical values in this table correspond to probabilities in actual binomial tables. See for example, Table 3 on page 614 of the IAAO Textbook, **Property Appraisal and Assessment Administration**.

By converting probabilities to critical values, we are able to reject the normal distribution hypothesis only when we can be at least 95% sure that a given distribution is not normal. For example, if a sample of 14 sales had four ratios above the mean and ten below, we would not be sufficiently confident to reject the normal distribution hypothesis. We would conclude that the sample did not deviate sufficiently from a normal distribution to support a non-normal conclusion. However, the actual probability of normality is only 9% in this case. In other words, in this and other tests of normality we are assuming normality. We will not conclude otherwise unless the evidence for the alternative conclusion is overwhelming and the chance of incorrectly concluding that we do ***not*** have a normal distribution is less than 5%.

Sample normality does not necessarily indicate population normality. In fact, much assessment literature specifically ascribes non-normal distributions to assessment ratio populations. However, even in non-normal populations, the central limit theorem holds that the distribution of means is nearly normal. This supposition requires only that there be a random and normally distributed probability of occurrence of a different mean if a second sample were selected. The second mean should be normally distributed in reference to the initial sample mean.

EDUCATION AND ASSISTANCE

The State Tax Commission plans and conducts an ongoing series of instructional workshops designed to help assessment personnel understand and utilize the statistical methods presented in this manual. In addition, computer programs have been designed to eliminate much of the drudgery involved in the computations. Complete programs are currently available for the following systems:

AS/400
IBM PC (or compatible)
with Lotus 1-2-3, version 5.0 or higher, or Excel

Technical Support Bureau personnel should be contacted for assistance in using the IBM ratio study program. For more information on the spreadsheet ratio study templates, contact Alan Dornfest at 334-7742.

THE RATIO STUDY AS AN APPRAISAL TOOL

The ratio study originally started as a tool to assist the appraisal process. With time it has become much better known as a method of measuring appraisal performance for the purpose of equalization. This perceived concept however, has by no means limited the usefulness of the statistics in appraisal work.

We commonly think of ratio studies as measuring level, uniformity, and regressivity. The exciting truth is that they can help with:

** identification of general assessment needs;
** identification of specific assessment needs;
** development of trend factors and time adjustments;
** measurement of appraiser performance;
** identification of assessment biases such as: class, location, construction type, age, value;
** defense of property values and appraisal technique in appeals;
** and of course, level, uniformity, and regressivity of assessments.

In addition to these pure uses of the ratio study, the statistics involved may be employed to help with the development of depth tables, economic rent and expenses, land schedules, depreciation tables, local cost modifiers, correlation of value estimates, etc.

In appraising we estimate a value for which a property would most probably sell under some specified conditions. Often in figuring this value we will process as many as three approaches to value with three different value indications; then, through a process known as correlation, we determine our final estimate.

In the past the appraiser often has chosen one value over another, or some combination of values. This often has been by the seat-of-the-pants or some other even worse method. However, by performing ratio studies using each of the cost, income, and market approaches, plus possibly a combination figure, you can study the results and know the best appraisal approach and value estimate.

Another example of using statistics in appraisal would be in analyzing what is economic rent. First, array the data and figure the mean, median, and mode of the group. Remember we don't want an average rent that might not reflect any property, we want a typical rent. Somewhere near your mean, median, and mode is the answer. By using uniformity statistics such as the COV and COD against several of your best estimates, you can arrive at a statistically supported number for economic rent.

The big key to meaningful results is the proper stratification of data into groups with a lot of similarity. The results you get from studying these groups are very valuable to assessment work and consequently increase the equity of the property tax.

2007 - 2008 RATIO STUDY STANDARDS

1. **Assessment Level**

The overall assessment level within any category of property will be considered to be market value provided the applicable ratio study measure of central tendency is within ±10% of 100%. This test will be considered met in the 2007 and 2008 ratio studies if the **median** based 90% (two tailed) confidence interval includes some part of the range between 90% and 110%.

For the 2007 and 2008 ratio studies, 80% confidence intervals will be computed and categories which pass using a 90% confidence interval, but fail using an 80% confidence interval will be noted. Final compliance status will not be determined using 80% confidence intervals until 2009 (the third ratio study under the revised system being implemented in 2007).

The following Chart shows examples of lowest and highest acceptable sample means given various sample sizes and standard deviations:

Proving Non-Compliance with 95% Confidence
Effect of Sample Size and Uniformity

Sample Size	Standard Deviation %	1-tail 1-score	Standard Error %	Lowest Acceptable Mean (%)	Highest Acceptable Mean (%)	Sample Size	Standard Deviation %	1-tail 1-score	Standard Error %	Lowest Acceptable Mean (%)	Highest Acceptable Mean (%)
5	5	2.132	4.77	85.23	114.77	**50**	5	1.645	1.16	88.84	111.16
5	10	2.132	9.53	80.47	119.53	**50**	10	1.645	2.33	87.67	112.33
5	15	2.132	14.30	75.70	124.30	**50**	15	1.645	3.49	86.51	113.49
5	20	2.132	19.07	70.93	129.07	**50**	20	1.645	4.65	85.35	114.65
5	30	2.132	28.60	61.40	138.60	**50**	30	1.645	6.98	83.02	116.98
5	40	2.132	38.14	51.86	148.14	**50**	40	1.645	9.31	80.69	119.31
10	5	1.833	2.90	87.10	112.90	**100**	5	1.645	0.82	89.18	110.82
10	10	1.833	5.80	84.20	115.80	**100**	10	1.645	1.65	88.36	111.65
10	15	1.833	8.69	81.31	118.69	**100**	15	1.645	2.47	87.53	112.47
10	20	1.833	11.59	78.41	121.59	**100**	20	1.645	3.29	86.71	113.29
10	30	1.833	17.39	72.61	127.39	**100**	30	1.645	4.94	85.07	114.94
10	40	1.833	23.19	66.81	133.19	**100**	40	1.645	6.58	83.42	116.58
20	5	1.729	1.93	88.07	111.93	**500**	5	1.645	0.37	89.63	110.37
20	10	1.729	3.87	88.13	113.87	**500**	10	1.645	0.74	89.26	110.74
20	15	1.729	5.80	84.20	115.80	**500**	15	1.645	1.10	88.90	111.10
20	20	1.729	7.73	82.27	117.73	**500**	20	1.645	1.47	88.53	111.47
20	30	1.729	11.60	78.40	121.60	**500**	30	1.645	2.21	87.79	112.21
20	40	1.729	15.46	74.54	125.46	**500**	40	1.645	2.94	87.06	112.94
30	5	1.699	1.55	88.45	111.55						
30	10	1.699	3.10	86.90	113.10						
30	15	1.699	4.65	85.35	114.65						
30	20	1.699	6.20	83.80	116.20						
30	30	1.699	9.31	80.69	119.31						
30	40	1.699	12.41	77.59	122.41						

Note: The preceding chart is provided as an indicator of compliance ranges, which actually will be based on medians and therefore cannot be mathematically predicted.

2. Assessment Uniformity

a. **Coefficient of Dispersion** (COD):

Uniformity is considered adequate if this measure does not exceed 15% in improved residential property categories. Dispersion of up to 20% is allowed in vacant property categories, manufactured housing, and commercial property categories.

b. **Coefficient of Variation** (COV):

Assessment uniformity is considered adequate if this measure does not exceed 20% in improved residential property categories. In vacant property categories, manufactured housing, and commercial property categories, variation of up to 25% is acceptable.

c. **Price-related Differential** (PRD):

Results in the 0.98 to 1.03 range are considered satisfactory.

Note: With the exception of agricultural land categories, measurements of uniformity of less than 5% are considered questionable, probably indicating invalid sample results. Results in this range should not be considered meaningful indicators of assessment uniformity.

STATE TAX COMMISSION EQUALIZATION PROCEDURE

The 2007 ratio study done by the State Tax Commission for equalization purposes is to be completed in March 2008. Prior to completion, consulting appraisers are to review each study to ensure that the data has been properly categorized and proper adjustments made. Upon completion, each county assessor is notified of the compliance status and complete statistical analysis of each category of property tested.

Each category found to be out of compliance with assessment level standards is subject to additional testing with a follow-up study using calendar year 2007 sale prices time-adjusted to January 1, 2008. These sale prices are compared against 2008 assessments as soon as these are available.

The results of the follow-up study are subject to corroboration using sales occurring through June, 2008. If the follow-up study is considered valid and representative, and indicates that adjustments made by the assessor have resulted in compliance with assessment level standards, the category is considered in compliance and no equalization recommendations are made to the State Tax Commission in 2008.

If a valid and representative follow-up study indicates non-compliance with assessment level standards, equalization recommendations will be developed on the basis of this follow-up study. If the follow-up study cannot be done or is otherwise considered invalid or non-representative, equalization recommendations may be made on the basis of the original ratio study or a corroborative study.

The State Tax Commission may delay implementation of any equalization adjustment for one year, if there is reason to question the representativeness of the original ratio study.

Prior to the meeting of the State Tax Commissioners as a Board of Equalization beginning on the second Monday of August each year, the State Tax Commission staff will prepare equalization recommendations for each county for categories of property still considered out of compliance with assessment level standards, in accordance with the following general procedure:

STAFF POSITION	**FUNCTION**
Property Tax Policy Supervisor	Presents statistical information from Ratio Study. Compares results to Ratio Study Standards and makes recommendations for equalization.
Consulting Appraiser	Presents judgment and opinion concerning the assessment conditions and the sample used for the Ratio Study. May concur with statistically based recommendation or may present substitute recommendation, based on additional information or findings regarding validity or representativeness of the ratio study or any follow-up study.
Division Administrator	Reviews all information and meets with Consulting Appraiser and Property Tax Policy Supervisor to decide on final recommendation to be submitted to State Board of Equalization.

Staff recommendations are not finalized until information has been compiled for all counties and categories subject to ratio study analysis.

The preceding process occurs during the period from March through July. Other than the Commissioner who supervises the Ad Valorem section, no State Tax Commissioner is involved in this staff recommendation procedure.

The Division Administrator finalizes staff recommendations for the State Board of Equalization and dates are established for the State Tax Commission to hear these recommendations during the State Board of Equalization meeting. Counties with equalization recommendations are then notified of the specific recommendations to be presented. County officials are invited to be present at a formal hearing. At this hearing the Division Administrator presents the staff recommendations and the Property Tax Policy Supervisor presents supportive information regarding these recommendations. County officials then have an opportunity to present any additional pertinent information for consideration.

The State Tax Commissioners take all recommendations under advisement and issue equalization decisions by the fourth Monday of August. Decisions are reached by voting commissioners, with the Commissioner who supervises the County Support Division participating in a non-voting, advisory capacity. Staff members are not present during final decision meetings.

Provided there is sufficient time to give affected property owners legally required notice and appeal rights, at any time during this process, until State Tax Commission (not staff) decisions are finalized, county commissioners may request permission from the State Tax Commission to reconvene the County Board of Equalization to change values to comply with ratio study standards.

Case I: Unacceptable Level

This situation can occur only if the 90% confidence interval around the sample median ratio fails to overlap any of the 90% - 110% range.

If unacceptable level is determined for a category for which the ratio study sample is considered non-representative or invalid, the county will be notified of non-complying status, but **no** trending recommendation will be prepared or submitted to the State Board of Equalization. If the level of assessment can be ascertained despite an inadequate follow-up ratio study, adjustment recommendations may be prepared and submitted to the State Board of Equalization.

The following procedures use the timeline for the 2007 equalization program (2007 ratio studies and 2008 Board of Equalization).

If unacceptable level is determined for a category for which the sample is considered representative (or a category that was **not** in compliance when last studied, but for which the sample had been considered non-representative), a trending recommendation will be prepared and submitted to the State Board of Equalization, unless:

a. the county can prove that adjustments equal to those about to be ordered have already been made, or

b. market value changes, demonstrated by as many new sales as are available through June, 2008, prove that the adjustments are no longer necessary, or

c. a validated follow-up ratio study shows that 2007 assessments are in compliance.

Each follow-up ratio study must be validated using procedures outlined in steps 1 through 4 below. The procedure in step 1 may also be used to demonstrate market changes, which may indicate compliance despite non-complying results in the original and follow-up ratio studies. Ratio study validation steps follow:

1.) Sales from October, 2007 through June, 2008 compared to 2008 assessments. These sales must be time-adjusted to January 1, 2008.

2.) A statement indicating trends or procedures used to correct values, including the number of parcels or identification of areas adjusted.

3.) A list of assessed values before and after application of adjustments. This list must be created for a randomly selected sample of non-selling properties within the areas adjusted within the non-complying category. The list must show the percent change between 2007 and 2008 values for each property selected and must show a total percent change for the entire random sample.

4.) A comparison of 2007 and 2008 total assessed value for the non-complying category.

If adequate proof is provided that county adjustments to non-complying categories have resulted in a median level of assessment that indicates compliance with assessment level standards, state adjustments will not be recommended. If, however, county adjustments alter assessment level, but do **not** produce results which comply with level standards, modified recommendations will be developed and submitted to the State Board of Equalization. If state ordered trending is necessary, the following procedure is used for staff recommendations:

1. Trending is to be by category (except as outlined in item 3 below) based on the median assessment ratio calculated for the category.

2. The trending factor is computed by dividing the chosen measure of level **into** 100.

3. Provided there is sufficient time to give affected property owners legally required notice and appeal rights, at any time during this process prior to State Board of Equalization action, a county may request that its board of equalization be reconvened for the purpose of trending by area or sub-category rather than by category of property. All new values must be submitted to the State Tax Commission for review.

4. Any category trended by the state in a given year will be considered in compliance with assessment level standards for that year, once the trend is applied.

Case II: Unacceptable Uniformity.

The county is notified of any category with unacceptable uniformity, but equalization action is not indicated in these cases.

RATIO STUDY REPRESENTATIVENESS

As has been discussed elsewhere, there are three distinct purposes for state ratio studies:

1. They are used to provide information to assist county assessors in their annual assessment programs.
2. They are used to evaluate uniformity to determine any potential need for the State Tax Commission to complete the county appraisal program.
3. They are used to assist the State Tax Commission in its task of equalizing assessments by category of property statewide.

The State Tax Commission is involved in the equalization process for several reasons including the following:

1. To ensure compliance with a federal requirement under the 4R Act for centrally assessed railroad property to be at (within ±5%) the same level as certain commercial and industrial properties.
2. To ensure that taxpayers in joint taxing districts are assessed at the same level regardless of their county of residence.
3. To ensure that taxpayers in different categories of property within one district are assessed equitably.
4. To maintain general equity between all centrally assessed property and locally assessed property.

To accomplish these tasks, it is critical that the state board of equalization be given the most reliable and credible information available regarding assessment level.

Review of assessed values and changes in assessed values of non-selling properties is one way of monitoring the representativeness of ratio study information.

DEFINITIONS

All terms are defined in accordance with their usage in this report.

Arithmetic Mean (average) ($\overline{A/S}$):

The result of dividing the sum of ratios in a series by n, the number of ratios in a sample. A measure of central value. Simple to compute, but may be disproportionately influenced by extreme ratios. Also known as: unweighted mean, mean.

$$\text{Mean} = \frac{\sum A/S}{n} = \overline{A/S}$$

Where A/S = each individual ratio and n = the number of ratios in a sample

Array:

An ordered series of ratios from low to high or high to low.

Binomial Test:

A procedure for determining whether ratios follow a normal distribution. Used if fewer than 100 sales are available in a sample.

Category:

Category means those types of property defined by numbers given in rule 130 and indicated on the county abstract of valuation.

A category is a specific type of property represented numerically on the county abstract of valuation.

The term also refers to the combinations typically used in the ratio study. Example: 20/41, urban residential lots with improvements.

Chi-Square Test:

A test by which the closeness of fit of sample data to a normal distribution may be determined.

$$\chi^2 = \frac{\Sigma(O-E)^2}{E} \quad \text{where}$$

χ^2 is the chi-square statistic,
O is the observed frequency, and
E is the expected frequency.

Confidence Interval:

An interval or range computed from a sample. This interval enables estimation of a population mean or median with a known degree of error. The bounds of the confidence interval are known as the upper confidence limit (UCL) and the lower confidence limit (LCL).

Dispersion, Coefficient of (COD):

The primary measure of assessment uniformity. It is based on the median and expresses uniformity in terms of the average difference between each ratio and the median. The COD is shown as a percent of the median.

Frequency Distribution:

An arrangement of ratios that groups this data to show how often given ratio ranges occur. (see: Histogram)

Geometric Mean:

A measure of assessment level determined by multiplying all of the values in a series together and then taking the "n"th root of this product.

$$\text{geometric mean} = \left(A_1/S_1 \; * \; A_2/S_2 \; * \; A_3/S_3 \; * \; \ldots \; A_n/S_n \right)^{1/n}$$

where A_n/S_n represents each ratio in the sample;
and n = the number of ratios.

Histogram:

A pictorial representation of a frequency distribution. (see: Frequency Distribution)

Hypothesis:

In inferential statistics this is a statement about which the truth or validity is to be tested. The usual procedure is to state what one chooses to accept in the absence of sufficient evidence to the contrary (the statement is called the "null hypothesis"), specify the relationship or statement to be proved (termed the "alternative hypothesis"), and analyze the available data to determine whether the null hypothesis can be rejected (and hence the alternative hypothesis accepted) at some confidence level.

Level:

A general expression of the overall relationship between assessed values and sales prices. Measured by determining:

1. Arithmetic mean
2. Median
3. Geometric mean
4. Weighted mean

Mean: (see: Arithmetic Mean or Weighted Mean or Geometric Mean)

Median $(\widetilde{A/S})$:

The middle ratio in an array. The rank of the ratio to be selected can be found by:

median rank = .5(n) + .5, where n = the number of ratios.

Also, equivalent to the 50th percentile.
(see: Dispersion, Coefficient of)

Mode:

The mode is the most frequently occurring number in a sample. If two or more numbers occur with equal frequency and no other number is present in greater frequency, there will be more than one mode.

Normal Distribution:

A type of frequency distribution which is symmetrical and approximately bell-shaped.

Population:

The group of all items or properties in a particular category from which a sample is drawn.

Ratio (A/S):

The result of dividing the assessed value of any property by its sale price. The answer is usually multiplied by 100 to be expressed as a percent.

Price-related Differential (PRD):

This statistic measures the treatment of property in relation to high or low value. Tendencies to value high priced property disproportionately with regard to low priced property can be identified.

The PRD is calculated by finding a weighted mean ratio by dividing the sum of the assessed value for all sales by the sum of the sale prices for all sales. This quotient is then divided into the unweighted mean ratio, with the result being the PRD.

Reliability:

Any measurement of the likelihood that sample results equal population results. Specifically, an attempt to determine whether the average assessment level measured using a ratio study sample corresponds to the true overall average assessment level on all properties in any category.

Weighted Mean:

An average ratio derived from the total assessed value and total sale price in an entire sample. This measure is subject to distortion if price-related assessment bias exists. (See: PRD).

Sample:

Sample means the sales which will be subject to ratio study analysis to reach a conclusion or make a recommendation relative to an abstract category of property in a county or in a specific area within a county.

Standard Deviation (s):

A statistical measure of the spread or distance of ratios from the mean in a sample.

$$s = \sqrt{\frac{\sum (A_i / S_i - (\overline{A/S})^2}{(n-1)}}$$

Where:

$\sum$	means "the sum of";
n	is the number of sales in the sample;
A_i/S_i	represents each individual ratio;
$\overline{A/S}$	is the mean ratio.

Standard Error of the Mean:

A statistic that indicates the probable magnitude of difference between a result (the arithmetic mean) obtained from a sample and the actual result if measured for a population as a whole.

$$SE_{\overline{A/S}} = \frac{\left(\frac{s}{\sqrt{n}}\right)}{\overline{A/S}}$$

where $SE_{A/S}$ = standard error of the mean $(\overline{A/S})$,

s = standard deviation,
and n = number of ratios in the sample.

(see: Standard Deviation)

Uniformity:

An expression of the equity of property taxes to the taxpayers within any given category of property. Determined by the following measurements:

1. Coefficient of Dispersion (COD)
2. Coefficient of Variation (COV)
3. Price-related Differential (PRD)

Variation, Coefficient of:

A measure of the spread of sample ratios from the mean expressed as a percent of the mean. An expression of the standard deviation in terms relative to the mean.

$$COV = \left(\frac{s}{\overline{A/S}}\right)*100$$

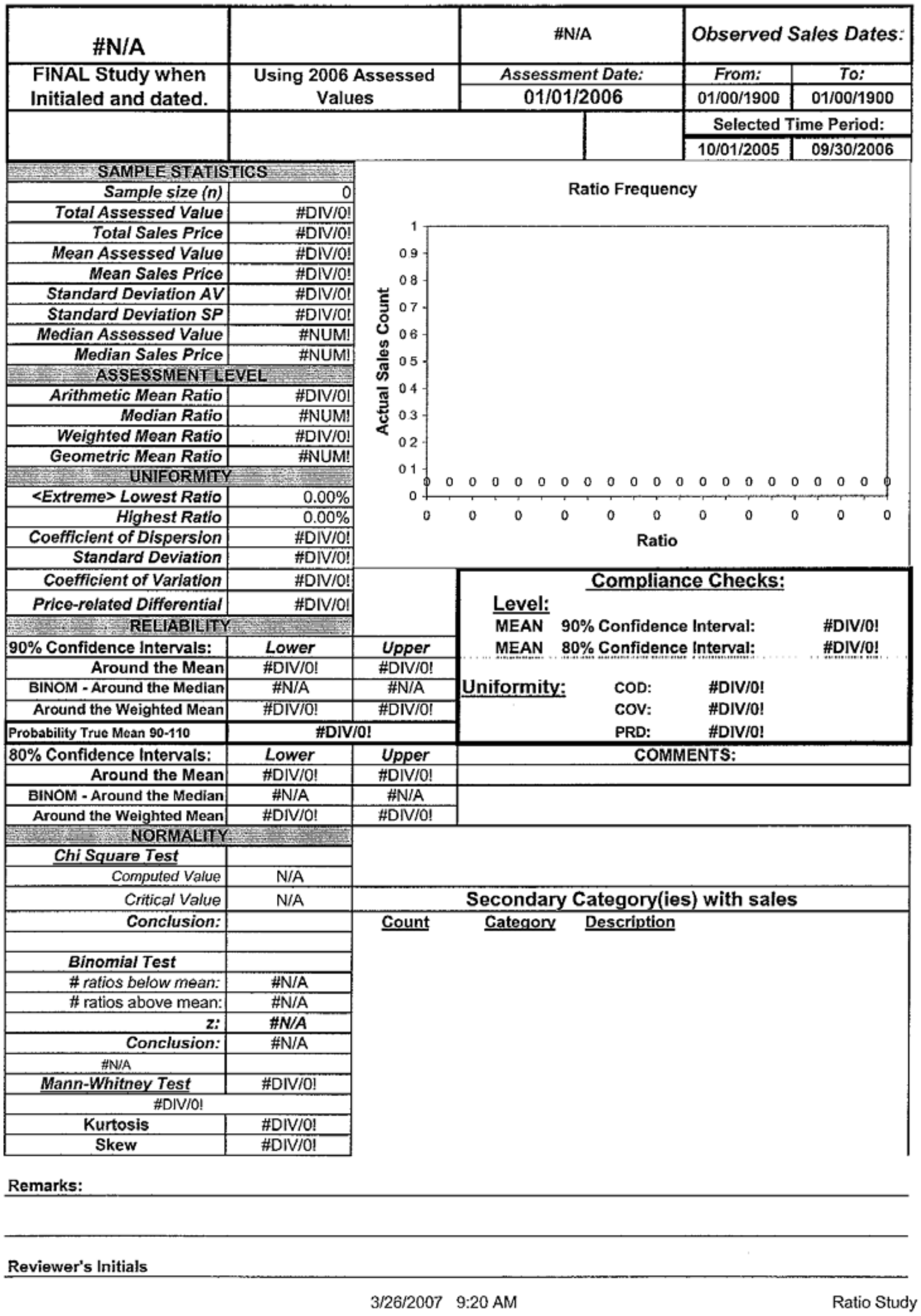
where s = standard deviation

$\overline{A/S}$ = the arithmetic mean.

(see: Standard Deviation)

APPENDICES

APPENDIX I a


Ratio Frequency
Actual Sales Count
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Ratio

#N/A		#N/A	*Observed Sales Dates:*	
FINAL Study when Initialed and dated.	Using 2006 Assessed Values	*Assessment Date:* 01/01/2006	*From:* 01/00/1900	*To:* 01/00/1900
			Selected Time Period:	
			10/01/2005	09/30/2006

SAMPLE STATISTICS		
Sample size (n)	0	
Total Assessed Value	#DIV/0!	
Total Sales Price	#DIV/0!	
Mean Assessed Value	#DIV/0!	
Mean Sales Price	#DIV/0!	
Standard Deviation AV	#DIV/0!	
Standard Deviation SP	#DIV/0!	
Median Assessed Value	#NUM!	
Median Sales Price	#NUM!	
ASSESSMENT LEVEL		
Arithmetic Mean Ratio	#DIV/0!	
Median Ratio	#NUM!	
Weighted Mean Ratio	#DIV/0!	
Geometric Mean Ratio	#NUM!	
UNIFORMITY		
<Extreme> Lowest Ratio	0.00%	
Highest Ratio	0.00%	
Coefficient of Dispersion	#DIV/0!	
Standard Deviation	#DIV/0!	
Coefficient of Variation	#DIV/0!	
Price-related Differential	#DIV/0!	
RELIABILITY		
90% Confidence Intervals:	*Lower*	*Upper*
Around the Mean	#DIV/0!	#DIV/0!
BINOM - Around the Median	#N/A	#N/A
Around the Weighted Mean	#DIV/0!	#DIV/0!
Probability True Mean 90-110	#DIV/0!	
80% Confidence Intervals:	*Lower*	*Upper*
Around the Mean	#DIV/0!	#DIV/0!
BINOM - Around the Median	#N/A	#N/A
Around the Weighted Mean	#DIV/0!	#DIV/0!
NORMALITY		
Chi Square Test		
Computed Value	N/A	
Critical Value	N/A	
Conclusion:		
Binomial Test		
# ratios below mean:	#N/A	
# ratios above mean:	#N/A	
z:	*#N/A*	
Conclusion:	#N/A	
#N/A		
Mann-Whitney Test	#DIV/0!	
#DIV/0!		
Kurtosis	#DIV/0!	
Skew	#DIV/0!	

Compliance Checks:

Level:

MEAN	90% Confidence Interval:	#DIV/0!
MEAN	80% Confidence Interval:	#DIV/0!

Uniformity:

COD:	#DIV/0!
COV:	#DIV/0!
PRD:	#DIV/0!

COMMENTS:

Secondary Category(ies) with sales

Count	Category	Description

Remarks:

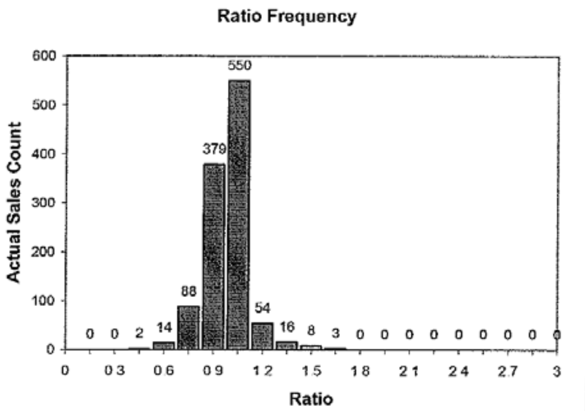
Reviewer's Initials

3/26/2007 9:20 AM Ratio Study

Appendix I b

COUNTY		Improved Urban Res	*Observed Sales Dates:*	
FINAL Study when Initialed and dated.	Using 2006 Assessed Values	*Assessment Date:* 01/01/2006	*From:* 10/01/2005	*To:* 09/01/2006
Sales Price is Time Adjusted	Inflationary Monthly Time Adjustment : ==>>	1.50%	Selected Time Period: 10/01/2005	 09/30/2006

SAMPLE STATISTICS	
Sample size (n)	1,114
Total Assessed Value	$123,819,552
Total Sales Price	$137,680,433
Mean Assessed Value	$111,149
Mean Sales Price	$123,591
Standard Deviation AV	$52,150
Standard Deviation SP	$59,885
Median Assessed Value	$95,605
Median Sales Price	$108,150
ASSESSMENT LEVEL	
Arithmetic Mean Ratio	90.83%
Median Ratio	91.67%
Weighted Mean Ratio	89.93%
Geometric Mean Ratio	89.89%
UNIFORMITY	
Lowest Ratio	33.88%
Highest Ratio	162.37%
Coefficient of Dispersion	10.00%
Standard Deviation	12.98%
Coefficient of Variation	14.29%
Price-related Differential	1.01


Ratio Frequency

Compliance Checks:

Level:

MEDIAN	90% Confidence Interval:	Passed
MEDIAN	80% Confidence Interval:	Passed

Uniformity:

COD:	Very Good
COV:	Very Good
PRD:	No Observed Bias

RELIABILITY		
90% Confidence Intervals:	*Lower*	*Upper*
Around the Mean	90.19%	91.47%
Around the Median	91.08%	92.39%
Around the Weighted Mean	89.20%	90.67%
Probability True Mean 90-110	98.30%	
80% Confidence Intervals:	*Lower*	*Upper*
Around the Mean	90.33%	91.33%
Around the Median	91.19%	92.26%
Around the Weighted Mean	89.36%	90.51%

COMMENTS:

NORMALITY	
Chi Square Test	
Computed Value	2230.96
Critical Value	12.59
Conclusion:	*Non-Normal*
Binomial Test	
# ratios below mean:	
# ratios above mean:	
z:	
Conclusion:	
*i.e., Insufficient evidence of Non-Normality	
Mann-Whitney Test	66.97141
Value Related Inequity	
Kurtosis	4.16
Skew	Positive

No data 'lost' to trim!
Sample Size is greater than 100 cases.
Trimming would NOT alter Statistics Significantly.

Secondary Category(ies) with sales

Count	Category	Description
1114	2041	Improved Urban Res

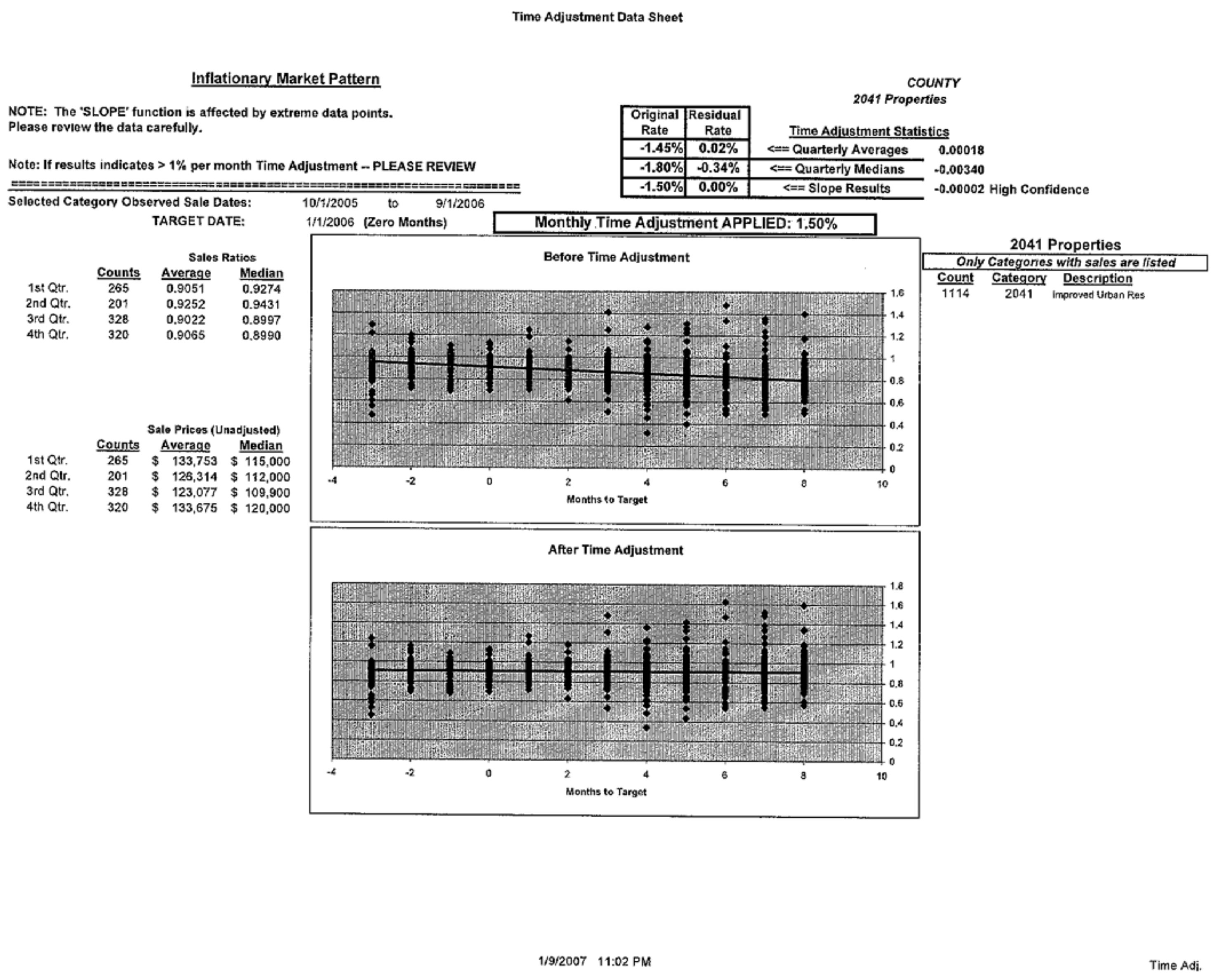
Remarks:

Reviewer's Initials

1/9/2007 11:02 PM

Ratio Study

Appendix II


Before Time Adjustment
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-4
-2
0
2
4
6
8
10
Months to Target
After Time Adjustment
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-4
-2
0
2
4
6
8
10
Months to Target

Time Adjustment Data Sheet

Inflationary Market Pattern

COUNTY

2041 Properties

**NOTE: The 'SLOPE' function is affected by extreme data points.
Please review the data carefully.**

Note: If results indicates > 1% per month Time Adjustment -- PLEASE REVIEW

Original Rate	Residual Rate	Time Adjustment Statistics	
-1.45%	0.02%	<== Quarterly Averages	0.00018
-1.80%	-0.34%	<== Quarterly Medians	-0.00340
-1.50%	0.00%	<== Slope Results	-0.00002 High Confidence

Selected Category Observed Sale Dates: 10/1/2005 to 9/1/2006

TARGET DATE: 1/1/2006 **(Zero Months)**

Monthly Time Adjustment APPLIED: 1.50%

Sales Ratios

	Counts	Average	Median
1st Qtr.	265	0.9051	0.9274
2nd Qtr.	201	0.9252	0.9431
3rd Qtr.	328	0.9022	0.8997
4th Qtr.	320	0.9065	0.8990

Sale Prices (Unadjusted)

	Counts	Average	Median
1st Qtr.	265	$ 133,753	$ 115,000
2nd Qtr.	201	$ 126,314	$ 112,000
3rd Qtr.	328	$ 123,077	$ 109,900
4th Qtr.	320	$ 133,675	$ 120,000

2041 Properties

Only Categories with sales are listed

Count	Category	Description
1114	2041	Improved Urban Res

1/9/2007 11:02 PM

Time Adj.

APPENDIX III a

SALES DATA RECORD

Study 1: Improved Residential (20/41)

	($) Assessor's Market Value				
Sale #	**Land**	**Imp.**	**Total**	**Sales Price ($)**	**Ratio(%)**
1	5,995	23,980	29,975	54,500	55.00
2	11,036	44,144	55,180	89,000	62.00
3	16,570	48,030	64,600	95,000	68.00
4	5,140	14,740	19,880	28,000	71.00
5	14,001	55,047	69,048	95,900	72.00
6	4,736	18,944	23,680	32,000	74.00
7	6,080	24,320	30,400	40,000	76.00
8	8,453	33,812	42,265	53,500	79.00
9	3,910	15,640	19,550	23,000	85.00
10	4,150	15,582	19,732	20,771	95.00
11	5,300	21,200	26,500	25,000	106.00
12	15,000	13,200	28,200	20,000	141.00

Study 2: Residential Land (20)

Sale #	Market Value	Sales Price ($)	Ratio(%)
1	6,400	8,000	80.00
2	10,080	12,000	84.00
3	9,660	10,500	92.00
4	3,906	4,200	93.00
5	5,820	6,000	97.00
6	7,425	7,500	99.00
7	11,330	11,000	103.00
8	9,360	9,000	104.00

APPENDIX III b1

Worksheet #2

Frequency Distribution, Relative Frequency and Histogram

Instruction: (Refer to your Sales Data Record)

1. List number of ratios falling within each interval separately for each category.
2. Total frequencies for both categories.
3. Using the combined total column only, divide the number of ratios in each interval by the total number of ratios.
4. Convert each answer to a percent by multiplying by 100. This is the relative frequency.
5. Transfer all information from this worksheet to the histogram sheet.
6. Draw the histogram.

Frequency Distribution:

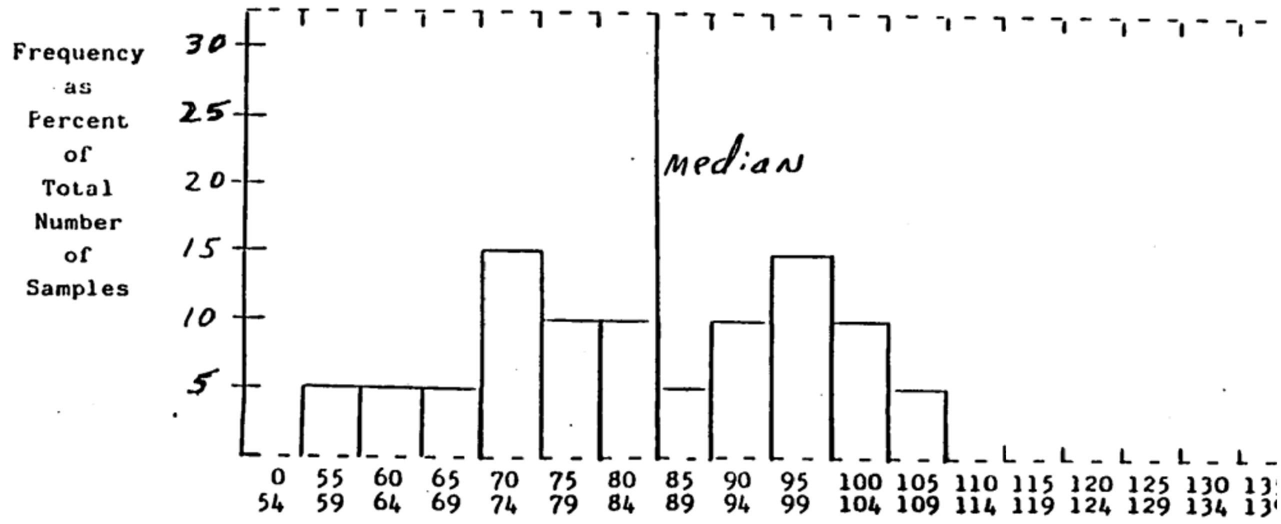
% Ratio Interval	**Frequency (Fi)**			**Frequency (%) (Fi/N) x 100**
	Imp. Resid. (20/41)	**Resid. Land (20)**	**Combined Total**	
Less than 55%	-	-	-	
55 – 59	1	-	1	5
60 – 64	1	-	1	5
65 – 69	1	-	1	5
70 – 74	3	-	3	15
75 – 79	2	-	2	10
80 – 84	-	2	2	10
85 – 89	1	-	1	5
90 – 94	-	2	2	10
95 – 99	1	2	3	15
100 – 104	-	2	2	10
105 – 109	1	-	1	5
140 – 144	1	-	1	5
Total	**XXXXXX**	**XXXXXX**	**20**	**100**

Appendix III b2

RATIO STUDY - SAMPLE FREQUENCY DISTRIBUTION AND HISTOGRAM

19__Ratio Study 19______Sales 19_____Assessments Summary for__________

Category	No. in Samples	Frequency of Ratios of Assessed Value to Sales Price/Appraisal Value																	
		0 54	55 59	60 64	65 69	70 74	75 79	80 84	85 89	90 94	95 99	100 104	105 109	110 114	115 119	120 124	125 129	130 134	1 1
Resid w/Imp.	12	—	1	1	1	3	2	—	1	—	1	—	1	—	—	—	—	—	—
Resid. land only	8	—	—	—	—	—	—	2	—	2	2	2	—	—	—	—	—	—	
Rural Inv.																			
Commercial																			
Other Rural with Improv.																			
Other Rural Land Only																			
Timber																			
Mobile Homes																			
TOTALS	20	—	1	1	1	3	2	2	1	2	3	2	1	—	—	—	—	—	—



APPENDIX III c1

Coefficient of Dispersion

Computation Worksheet #3

Purposes: Using the data from your Sales Data Record, complete the following charts and compute the coefficient of dispersion (COD) for each category of property.

Improved Residential Category (20/41)			
A	**B**	**C**	**D**
Sale #	**Ratio**	**Median**	**Difference between Median and Ratio**
1	55.00	75.00	20.00
2	62.00	75.00	13.00
3	68.00	75.00	7.00
4	71.00	75.00	4.00
5	72.00	75.00	3.00
6	74.00	75.00	1.00
7	76.00	75.00	1.00
8	79.00	75.00	7.00
9	85.00	75.00	10.00
10	95.00	75.00	20.00
11	106.00	75.00	31.00
12	141.00	75.00	66.00
Total:			**180.00**

Worksheet #3 (Continued)

Residential Land Category (20)			
A	**B**	**C**	**D**
Sale #	**Ratio**	**Median**	**Difference between Median and Ratio**
1	80.00	95.00	15.00
2	84.00	95.00	11.00
3	92.00	95.00	3.00
4	93.00	95.00	2.00
5	97.00	95.00	2.00
6	99.00	95.00	4.00
7	103.00	95.00	8.00
8	104.00	95.00	9.00
Total:			**54.00**

APPENDIX III c3

Worksheet #3 (Continued)

Steps for Above Charts:

1. In Column B, list ratios from Sales Data Record.

2. Determine the median ratio for each category and show this figure in Column C next to each ratio shown in Column B.

3. Subtract the median from Column B ratio. Ignore the sign (±) of the answer (this gives us the absolute value of the difference) shown in Column D.

4. Add up Column D for each chart and show the total. Treat all numbers as if they were positive.

5. Determine the average absolute deviation by dividing the total difference from Column D by n (the number of sales). (Use space provided below to show calculation for each category.)

$$\frac{\text{Total Difference}}{n} = \text{Average absolute deviation}$$

Imp. Resid. Category:

$$\frac{180.00}{12} = 15.00$$

Resid. Land Category:

$$\frac{54.00}{8} = 6.75$$

APPENDIX III c4

Worksheet #3 (Continued)

6. Divide your answers from step 5 by each median and multiply by 100 to find the Coefficient of Dispersion (COD) expressed as a percent. (Use space provided below.)

$$\frac{\text{Average Absolute Deviation}}{\text{median}} \text{ x } 100 = \text{COD}$$

Imp. Resid. Category:

$$\frac{15.00}{75.00} \text{ x } 100 = 20.00$$

Resid. Land Category:

$$\frac{6.75}{95.00} \text{ x } 100 = 7.11$$

Conclusions:

Compare uniformity in the two categories.

The improved residential category demonstrates somewhat poor uniformity, while there is excellent uniformity shown by the residential land sample.

APPENDIX III d1

Worksheet #4

Standard Deviation and Coefficient of Variation

Purpose: Using the data from your Sales Data Record, complete the following charts and compute the standard deviation and coefficient of variation for each category of property.

Improved Residential Category (20/41)				
A	**B**	**C**	**D**	**E**
	Ratio	**Mean Ratio**	**Difference**	**(Difference)^2**
Sale #	(A/S)*100	$\overline{A/S}$	$\left(A/S-\overline{A/S}\right)$	$\left(A/S-\overline{A/S}\right)^2$
1	55.00	82.00	-27.00	729.00
2	62.00	82.00	-20.00	400.00
3	68.00	82.00	-14.00	196.00
4	71.00	82.00	-11.00	121.00
5	72.00	82.00	-10.00	100.00
6	74.00	82.00	-8.00	64.00
7	76.00	82.00	-6.00	36.00
8	79.00	82.00	-3.00	9.00
9	85.00	82.00	3.00	9.00
10	95.00	82.00	13.00	169.00
11	106.00	82.00	24.00	576.00
12	141.00	82.00	59.00	3,481.00
Totals:	**984.00**			**5,890.00**

APPENDIX III d2

Worksheet #4 (Continued)

Improved Residential Category (20/41)				
A	**B**	**C**	**D**	**E**
	Ratio	**Mean Ratio**	**Difference**	**(Difference)^2**
Sale #	(A/S)*100	$\overline{A/S}$	$\left(A/S-\overline{A/S}\right)$	$\left(A/S-\overline{A/S}\right)^2$
1	80.00	94.00	-14.00	196.00
2	84.00	94.00	-10.00	100.00
3	92.00	94.00	-2.00	4.00
4	93.00	94.00	-1.00	1.00
5	97.00	94.00	3.00	9.00
6	99.00	94.00	5.00	25.00
7	103.00	94.00	9.00	81.00
8	104.00	94.00	10.00	100.00
Totals:	**752.00**			**516.00**

APPENDIX III d3

Worksheet #4 (Continued)

Procedure:

1. In Column B of each chart, list ratios from sales data record.

2. Total Column B ratios.

3. Divide each Column B total by n to get the mean ratio ($\overline{A/S}$). Show results below.

   Imp. Resid. mean $\frac{984}{12}$ = 82.00%

   Resid. Land mean $\frac{752}{8}$ = 94.00%

4. Show the mean ratio in Column C next to each Column B ratio.

5. Subtract the mean ratio from each Column B ratio and show the result in Column D.

6. Square each Column D result and show in Column E.

7. Total the numbers in Column E.

8. Divide the total of Column E by n-1. Show result below:

   Imp. Resid.: $\frac{\text{Col. E. Total}}{\text{n-1}} = \frac{5890}{11} = 535.45$

   Resid. Land: $\frac{\text{Col. E. Total}}{\text{n-1}} = \frac{516}{7} = 73.71$

   (This is known as the variance.)

9. Take the square root of the figure obtained in Step 8. This is the standard deviation. Show result below:

   Imp. Resid.: $\sqrt{535.45} = 23.14$
   Resid. Land: $\sqrt{73.71} = 8.59$

APPENDIX III d4

Worksheet #4 (Continued)

10. Divide the standard deviation by the mean (Step 3) and multiply your result by 100 to get the coefficient of variation.

$$\frac{s}{\overline{A/S}}\ x\quad 100 = COV$$

Imp. Resid.: $\frac{23.14}{82.00}$ x 100 = (COV) 28.22%

Resid. Land: $\frac{8.59}{94.00}$ x 100 = (COV) 9.13%

Conclusions:

1. Compare the COV's determined for each category.

   The COV for the improved residential category is considerably higher than for the residential land category. Excellent uniformity is indicated for the residential land, but improved residential uniformity is poor.

2. How do the COV's compare with the COD's you calculated on Worksheet #3?

   Results for both categories are higher than corresponding COD's.

APPENDIX III e

PRD
Worksheet #5

The PRD compares a weighted mean to the simple mean determined for the ratios. Index numbers greater than 1.0 indicate a tendency to favor higher priced properties, valuing them proportionately lower (lower ratios) than lower priced properties.

Procedure:

Use the data from the Sales Data Record to complete the following steps for each category of property.

Step		**Result**	
#	**Description of Step**	**Imp. Resid.**	**Res. Land**
1	Sum of Assessor's market values	429,010.00	63,981.00
2	Sum of sales prices	576,671.00	68,200.00
3	$\frac{\text{Step 1 result}}{\text{Step 2 result}} \times 100$ = weighted Mean ratio (%)	74.39	93.81
4	Sum of Ratios (%)	984.00	752.00
5	$\frac{\text{Sum of Ratios}}{n}$ = Mean ratio (%)	82.00	94.00
6	$\frac{\text{Mean Ratio}}{\text{Wtd. mean Ratio}}$ = PRD	1.10	1.00

Compare each PRD.

> The improved residential category PRD indicates a tendency to over-assess lower priced properties. Results on the residential land category show no tendency to favor either higher or lower priced properties.

APPENDIX III f

MEASUREMENTS OF UNIFORMITY

Shortcut Procedures

1. Coefficient of Dispersion (Resid. Land example).

    a. Sum of the ratios below the median in your array 349
    b. Sum of the ratios above the median in your array 403
    c. Subtract the sum in (a) from the sum in (b) 54
    d. Divide step (c) result by n 6.75
    e. Divide step (d) result by median; multiply by 100 7.11

2. Coefficient of Variation (Residential Land)

A Sale	B Ratio	C $(\text{Ratio})^2$
1	80.00%	6400
2	84.00%	7056
3	92.00%	8464
4	93.00%	8649
5	97.00%	9409
6	99.00%	9801
7	103.00%	10609
8	104.00%	10816
Total	752.00	71204

STEPS:

1. Square each ratio and show result in Column C.
2. Find the sum of Column B.
3. Find the sum of Column C. **71204**
4. Square the sum of Column B and divide the result by n.

$$\frac{(\text{Sum of Col. B})^2}{n} = \frac{565504}{8} = 70688$$

5. Subtract Step 4 result from Step 3 result 516
6. Divide the result in Step 5 by n-1 73.7143
7. Take the square root of your Step 6 answer 8.59
8. Divide Step 2 result by n to find the mean 94.00
9. Divide the standard deviation (Step 7) by the mean (Step 8) and multiply the result by 100.

$$\frac{\text{Standard Deviation}}{\text{Mean}} \times 100$$

$$\frac{8.59}{94.00} \times 100 = 9.13 = \text{COV}$$

Appendix III g

Using the Mann-Whitney test to determine equity between two property groups.

Mean Sale Price: $102,263

SP<mean Ratio	Rank	SP>mean Ratio	Rank	Sale Price	Assessed Value	Ratio $(\overline{A/S})$	Rank
		0.5174	1	$290,130	$150,127	0.5174	1
0.5294	2			$33,425	$17,694	0.5294	2
		0.5649	3	$286,500	$161,851	0.5649	3
		0.5953	4	$106,808	$63,580	0.5953	4
0.6175	5			$63,674	$39,320	0.6175	5
		0.6229	6	$243,525	$151,692	0.6229	6
0.6497	7			$71,625	$46,537	0.6497	7
		0.6592	8	$273,294	$180,156	0.6592	8
0.6683	9			$47,718	$31,890	0.6683	9
0.6713	10			$39,764	$26,692	0.6713	10
0.6729	11			$91,948	$61,875	0.6729	11
		0.6776	12	$119,375	$80,888	0.6776	12
0.7069	13			$91,326	$64,554	0.7069	13
0.7077	14			$77,025	$54,510	0.7077	14
0.7749	15			$58,750	$45,523	0.7749	15
0.7767	16			$74,960	$58,224	0.7767	16
0.7777	17			$56,760	$44,142	0.7777	17
0.7787	18			$58,464	$45,523	0.7787	18
0.7900	19			$56,476	$44,618	0.7900	19
		0.7996	20	$114,314	$91,409	0.7996	20
0.8033	21			$73,788	$59,272	0.8033	21
		0.8053	22	$247,750	$199,525	0.8053	22
		0.8054	23	$177,845	$143,236	0.8054	23
		0.8217	24	$112,930	$92,793	0.8217	24
		0.8240	25	$104,597	$86,191	0.8240	25
0.8287	26			$57,744	$47,850	0.8287	26
0.8423	27			$92,430	$77,855	0.8423	27
		0.8567	28	$135,000	$115,660	0.8567	28
0.8801	29			$56,220	$49,478	0.8801	29
0.8949	30			$40,291	$36,057	0.8949	30
0.9800	31			$43,785	$42,908	0.9800	31
		1.0323	32	$106,177	$109,602	1.0323	32
1.0480	33			$23,425	$24,549	1.0480	33
1.2834	34			$35,945	$46,132	1.2834	34
1.5049	35			$15,405	$23,183	1.5049	35
Total:	**422**		**208**				
Number:	**22**		**13**				
Average:	**19.2**		**16.0**				

U = [(22*13)+[22*(22+1)]/2]-422

$$U = \mathrm{n}_1 n_2 + \frac{\mathrm{n}_1(n_1+1)}{2} - \mathrm{R}_1$$

U = 117

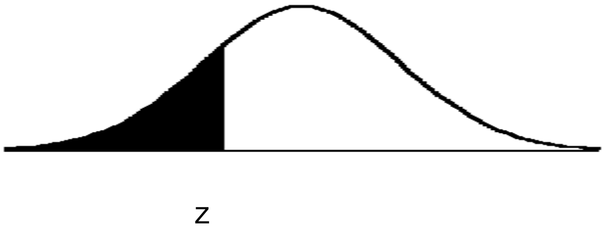
Z = [(117-((22-13)/2)][sqrt((22*13)*(22+13+1))/12]
Z = -0.88763

$$z = \frac{U - (n_1 n_2)/2}{\sqrt{(n_1 n_2)(n_1 + \mathrm{n}_2 + 1)/12}}$$

The difference is not significant; value related inequity cannot be proven.

Appendix IV a

Values of the Standard Normal Distribution Function



Values of the Standard Normal Distribution Function										
Z	0	1	2	3	4	5	6	7	8	9
-3.	.0013	.0010	.0007	.0005	.0003	.0002	.0002	.0001	.0001	.0000
-2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0126	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0238	.0233
-1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0300	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0570	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
- .9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
- .8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
- .7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
- .6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
- .5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
- .4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
- .3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
- .2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
- .1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
- .0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

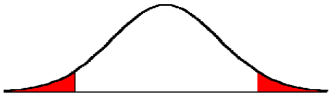
Appendix IV b

Values of the Standard Normal Distribution Function

Values of the Standard Normal Distribution Function										
Z	0	1	2	3	4	5	6	7	8	9
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	7995	.8023	.8051	.8078	.8106	.8133
9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9430	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9648	.9656	.9664	.9671	.9678	.9686	.9693	.9700	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9762	.9767
2.0	.9772	.9778	.9783	9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9874	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9990	.9993	.9995	.9997	.9998	.9998	.9999	.9999	1.0000

Appendix V

Table X Table of “Student’s” Distribution: Value of t

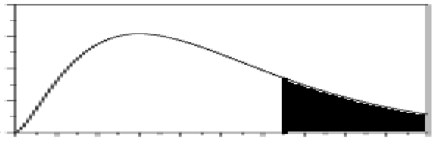


Degrees of Freedom	Probability												
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05	0.02	0.01	0.001
1	0.158	0.325	0.510	0.727	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	636.619
2	0.142	0.289	0.445	0.617	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.598
3	0.137	0.277	0.424	0.584	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	12.924
4	0.134	0.271	0.414	0.569	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	0.132	0.267	0.408	0.559	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	6.869
6	0.131	0.265	0.404	0.553	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.959
7	0.130	0.263	0.402	0.549	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	5.408
8	0.130	0.262	0.399	0.546	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	5.041
9	0.129	0.261	0.398	0.543	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.781
10	0.129	0.260	0.397	0.542	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.587
11	0.129	0.260	0.396	0.540	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.437
12	0.128	0.259	0.395	0.539	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	4.318
13	0.128	0.259	0.394	0.538	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
14	0.128	0.258	0.393	0.537	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
15	0.128	0.258	0.393	0.536	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	4.073
16	0.128	0.258	0.392	0.535	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
17	0.128	0.257	0.392	0.534	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
18	0.127	0.257	0.392	0.534	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
19	0.127	0.257	0.391	0.533	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
20	0.127	0.257	0.391	0.533	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
21	0.127	0.257	0.391	0.532	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.819
22	0.127	0.256	0.390	0.532	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.792
23	0.127	0.256	0.390	0.532	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.767
24	0.127	0.256	0.390	0.531	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.745
25	0.127	0.256	0.390	0.531	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.725
26	0.127	0.256	0.390	0.531	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.707
27	0.127	0.256	0.389	0.531	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.690
28	0.127	0.256	0.389	0.530	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
29	0.127	0.256	0.389	0.530	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.659
30	0.127	0.256	0.389	0.530	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
40	0.126	0.255	0.388	0.529	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
60	0.126	0.254	0.387	0.527	0.679	0.848	1.046	1.296	1.671	2.000	2.390	2.660	3.460
120	0.126	0.254	0.386	0.526	0.677	0.845	1.041	1.289	1.658	1.980	2.358	2.617	3.373
∞	0.126	0.253	0.385	0.524	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

This table is abridged from Table II of Fisher and Yates: Statistical Tables for Biological, Agricultural and Medical Research, published by Longman Group Ltd., London (previously published by Oliver & Boyd Ltd., Edinburgh) and by permission of the author and publishers.

Appendix VI

The Chi-Square Distribution



DF	Probability that Chi-Square value will be exceeded							
	.995	.990	.975	.950	.050	.025	.010	.005
1				.004	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	90.53	95.02	100.43	104.22
80	51.17	53.54	57.15	60.39	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	113.14	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	124.34	129.56	135.81	140.17

131. USE OF RATIO STUDY TO TEST FOR EQUALIZATION IN COUNTIES (RULE 131).

Section 63-109, Idaho Code. (3-30-07)

01. Equalization Ratio Study. Each year the State Tax Commission shall conduct a ratio study to assist in the equalization of assessments of property within and among the primary_categories established in Rule 130 of these rules. The ratio study shall be conducted in accordance with the "Standard on Ratio Studies" referenced in Rule 006 of these Rules. The annual ratio study shall test assessments as of January 1 of each year. Except when sales or appraisals must be added or deleted to improve representativeness, sales used will be those occurring within each county between October 1 of the year preceding the year for which assessments are to be tested and September 30 of the year for which assessments are to be tested. Each sale price is to be adjusted for time and compared to market value for assessment purposes for the year for which assessments are to be tested, to compute ratios to be analyzed. The State Tax Commission may use sales from extended time periods and may add appraisals when data is lacking. The State Tax Commission may delete sales when necessary to improve representativeness. The study shall be completed in February following the end of the period studied. The appropriate ratio study statistical measure of level shall be the median for equalization ratio studies conducted beginning October 1, 2006. (3-30-07)

02. Tested For Equalization. Beginning with the 2007 ratio study to be complete prior to the first Monday in April, 2008, categories to be tested for equalization purposes are the primary categories, described in Subsections 130.02 through 130.06 of these rules, provided adequate samples can be obtained. (3-30-07)

03. Follow Up Ratio Study. When the annual ratio study provided in Subsections 131.01 and 131.02 of this Rule, discloses that assessments in any primary category as described in Subsections 130.02 through 130.06 of these Rules are out of compliance with the equalization standards of this rule, the State Tax Commission shall conduct a follow-up ratio study. The follow-up ratio study shall test the assessments for January 1 of the year following the year tested by the annual ratio study and shall be based on property sales occurring during the calendar year immediately preceding that date. The State Tax Commission shall notify the county assessor of the results of the follow-up ratio study. The notice shall indicate whether any adjustments will be considered by the State Tax Commission at its next equalization meeting in August based on either the annual, or any follow-up ratio study, and the reason for the proposed adjustments.
(3-30-07)

04. Use of Ratio Study Results. The results of the annual ratio study or any follow-up ratio study shall be one source of information upon which the State Tax Commission may rely when testing assessments for equalization purposes under Section 63-109, Idaho Code. When the results of any ratio study on any primary category described in Subsections 130.02 through 130.06

of these rules, show, with reasonable statistical certainty as defined in Subsection 131.07 of this Rule, that the appropriate measure of level of any primary category studied is less than ninety percent (90%) or greater than one hundred ten percent (110%), the assessment of property within that primary category may be considered not equalized. When this occurs, the State Tax Commission may, at its annual meeting commencing on the second Monday in August, order the county auditor to adjust the value of all property in any primary category or subcategory included in the analysis conducted in an amount the State Tax Commission finds necessary to accomplish equalization of assessments of property. Within any primary category except as provided in Subsection 131.05 of this rule, adjustment will not be considered for any secondary category, described in Rule 510, 511, or 512 of these rules, that does not have at least one observation in the ratio study conducted for that primary category. (3-30-07)

05. Exception from Requirement for at Least One Observation for Use of Secondary Category in Adjusted Value Determination. Properties identified as secondary categories 10 and 31 rarely sell separately from farms and therefore do not appear in any ratio study. However, the level of assessment typically is similar to that of other rural residential property, including property in secondary categories 12, 15, 34, and 37. For any ratio study where there is an adjustment to be made to the assessed values in the residential designation, such adjustment shall be applied to any assessed value in secondary category 10, provided there is at least one observation (sale) of property identified in either secondary category 12 or 15. Such adjustment shall also be applied to any assessed value in secondary category 31, provided there is at least one observation (sale) of property identified in either secondary category 34 or 37.
(3-30-07)

06. Use of Alternate Ratio Study. When the follow-up ratio study required by Subsection 131.03 of this Rule does not measure the true assessment level, the State Tax Commission may consider adjustment based on the most recent annually conducted ratio study or other information relevant to equalization. If the State Tax Commission has reason to question the representativeness of the sample used in an annual or follow up ratio study conducted on any primary category, the State Tax Commission may delay implementation of any order to adjust property values until two successive years' ratio studies fail to produce an appropriate measure of level between ninety percent (90%) and one hundred ten percent (110%).
(3-30-07)

07. Submission of Additional Information. Any party may petition the State Tax Commission to consider any information or studies relevant to equalization. The petition shall include a description of the information to be presented and the petitioner's conclusions drawn from the information.
(4-5-95)

08. Reasonable Statistical Certainty. For the purposes of Rule 131 and equalization pursuant to Section 63-109, Idaho Code, "reasonable statistical certainty" that any primary category is not equalized shall mean that the appropriate measure of level determined by the ratio study for the primary category must be provably less than ninety percent (90%) or greater than one hundred ten percent (110%) of market value for assessment purposes. Such a determination shall occur if: (3-30-07)

a. The appropriate measure of level for the primary category(ies)being tested is less than ninety percent(90%)or greater than one hundred ten percent (110%)and a ninety percent(90%)two-tailed confidence interval around the appropriate measure of level fails to include ninety percent(90%)or one hundred ten percent(110%); or (3-30-07)

b. The appropriate measure of level for the primary category(ies) being tested has been less than ninety percent (90%) or greater than one hundred ten percent (110%) as determined by the most recent previous two (2) ratio studies on the primary category(ies) and an eighty percent (80%) two-tailed confidence interval around the appropriate measure of level fails to include ninety percent (90%) or one hundred ten percent (110%). No ratio study conducted prior to January, 2007 will be considered as one of the most recent previous two (2) ratio studies. (3-30-07)

09. Cross References. The primary categories are described in Subsections 130.02 through 130.06 of these Rules, and the secondary categories are described in Rules 510, 511, and 512 of these Rules. (3-30-07)

315. RATIO STUDIES - SCHOOL DISTRICTS (RULE 315).

Section 63-315, Idaho Code. (5-3-03)

01. Procedures for School District Ratio Studies. The ratio study conducted by the State Tax Commission to comply with the requirements of Section 63-315, Idaho Code, shall be conducted in accordance with the "Standard on Ratio Studies" referenced in Rule 006 of these rules. The following specific procedures will be used. (4-11-06)

a. Information on property sales, which meet the requirements of arm's length and market value sales, will be obtained and assembled into samples representing various categories of property and designations defined in Subsection 315.02 of this rule in each school district. Except when sales or appraisals must be added or deleted to improve representativeness, sales used will be those occurring within each school district between October 1 of the year preceding the year for which adjusted market value is to be computed and September 30 of the year for which adjusted market value is to be computed. Each sale price is to be adjusted for time and compared to market value for assessment purposes for the year for which adjusted market value is to be computed, to compute ratios to be analyzed. The State Tax Commission may use sales from extended time periods and may add appraisals when data is lacking. The State Tax Commission may delete sales when necessary to improve representativeness. (4-11-06)

b. A ratio will be determined for each sale by dividing the market value for assessment purposes of the property by the adjusted sale price or appraised value. (7-1-98)

c. A statistical analysis is to be conducted for the sales and any appraisals in each property designation defined in Subsection 315.02 of this rule in each school district and appropriate measures of central tendency, uniformity, reliability, and normality computed.

(4-11-06)

d. With the exception of any property designations with extended time frames or added appraisals, if fewer than five (5) sales and appraisals are available, no adjustment to the taxable value of the designation will be made. (7-1-98)

e. If there are five (5) or more sales and appraisals and it is determined with reasonable statistical certainty that the property designation is not already at market value for assessment purposes, an adjusted market value will be computed for the school district by dividing the taxable value for the year for which adjusted market value is to be determined by the appropriate ratio derived from the ratio study. The appropriate ratio to be used shall be the weighted mean ratio calculated from the sample for each designation, unless it can be clearly demonstrated that this statistic has been distorted by nonrepresentative ratios. In this case the median may be substituted:

(3-30-01)

f. Within each school district, adjusted market value or taxable value for each category of real, personal and operating property will be summed to produce the total adjusted market value for the school district. The school district taxable value will then be divided by this adjusted market value to produce the overall ratio of assessment in each school district. Statewide totals are to be calculated by compiling county totals. (7-1-98)

g. Urban renewal increment values will not be included in the taxable value or the adjusted market value for any school district. (7-1-98)

h. "Reasonable statistical certainty," that the property designation in question is not at market value for assessment purposes, is required. Such certainty is tested using ninety percent (90%) confidence intervals about the weighted mean or median ratios. If the appropriate confidence interval includes ninety-five percent (95%) or one hundred five percent (105%), there is not "reasonable statistical certainty" that the property designation is not at market value for assessment purposes. (3-30-01)

i. Categories of property subject to adjustment following the procedure outlined in this rule and ratio study designations from which measures of central tendency used for adjustments will be derived are:

Category	Property Category	Ratio Study Designation
41	Urban Residential Improvements	Residential
20	Urban Residential Land	Residential
37	Rural Residential Subdivision Improvements	Residential
15	Rural Residential Subdivision Land	Residential
34 & 40	Rural Residential Tract and Other Rural Improvements	Residential
12 & 18	Rural Residential Tracts and Other Lands	Residential
42	Urban Commercial Improvements	Commercial
21	Urban Commercial Land	Commercial
35 & 38	Rural Commercial Tract and Subdivision Improvements	Commercial
13 & 16	Rural Commercial Tracts and Subdivision Land	Commercial
46, 47, & 65	Manufactured Homes and Attachments	Residential
48 & 49	Manufactured Homes Declared to be Real Property	Residential
26	Residential Condominiums	Residential
27	Commercial Condominiums	Commercial

(4-11-06)

j. For all other property categories not contained in the list in Subsection 315.01.i. of this rule, adjusted market value will equal taxable value. (4-11-06)

k. "Appraisal" or "appraised value" refers to any State Tax Commission provided independently conducted property appraisal. (7-1-98)

02. Use of Property Designations. In computing the ratio for each school district, the State Tax Commission will designate property as residential, commercial, or manufactured housing and shall assign appropriate property categories defined in Rule 130 of these rules to these designations. For each school district, adjusted market value shall be computed by dividing the appropriate ratio ascertained for each of these designations into the sum of the taxable values for each category of property assigned to a designation. For the taxable value in any category to be included in said sum, at least one (1) observation (sale or appraisal) from that category must be present in the ratio study. If the ratio for any given designation in a school district indicates that the market value for assessment purposes cannot be determined with reasonable statistical certainty to differ from statutorily required market value, the taxable value shown on the school district abstract(s) required pursuant to Subsection 315.04 of this rule for each of the categories included in that designation shall be the adjusted market value for said designation for said school district. (4-11-06)

03. Assessor to Identify School Districts. Each county assessor will provide to the State Tax Commission the school district in which each sale submitted for the ratio study is located. (7-1-98)

04. Abstracts of Value by School District. Each county auditor shall provide to the State Tax Commission abstracts of the taxable value of all property within the portion of each school district in each county. These abstracts shall be submitted in the same manner and at the same time as provided for county abstracts of value. (7-1-98)

05. Urban Renewal Increment and Exemption to be Subtracted. The taxable value of each category of property within each school district shall not include the value that exceeds the value on the base assessment roll in any urban renewal district pursuant to Chapter 29, Title 50, Idaho Code, and shall not include the value of any exemption pursuant to Sections 63-602G, 63-602K, 63-602P, 63-602X, 63-602AA, 63-602BB, 63-602CC, and 63-602FF, I (4-11-06)